HE 215 : Nuclear & Particle Physics Course

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August 2018 Lectures

Outline I

- A Brief History of Particle Physics Discoveries
  - Standard Model
  - Hadron Colliders
  - Particle Physics - The Zoo
  - History of Universe
- Nuclear Physics
  - Liquid drop model or Semi-empirical mass formula
  - Nuclear Stability or β- Stability curve
  - Shell Model
  - Radioactivity
  - gamma decay
  - beta decay
  - Q-value of nuclear reaction
  - alpha decay
  - Nuclear Fission
  - Nuclear Fusion

References:

- Introduction to elementary particles by David Griffiths - 2nd edition - John Wiley & Sons
- Particles And Nuclei by Bogdan Povh, Klaus Rith, Christoph Scholz, Frank Zetsche - 6th edition Springer
- Introduction to high energy physics by Donald Perkins - Third edition Addison-Wesley
- Introduction to Nuclear and Particle Physics by Das & Ferbel - World Scientific
- Introductory Nuclear Physics by Krane K.S. - John Wiley & Sons
- Introduction to Elementary Particle Physics by Alessandro Bettini - Cambridge
- Particle Physics by Martin, B.R. & Shaw, G. - Wiley

For Particle Physics I will mostly follow
Introduction to Elementary Particles, 2nd Revised Edition by Griffiths

All the slides are available on google Drive link:

`https://goo.gl/vepcJi`

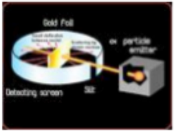
A Brief History of Particle Physics Discoveries

This is chapter 1 in Griffiths.

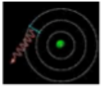
Looking Inside the Atom: e^-, p, and n

- 1897: J.J. Thomson discovers that the charge/mass ratio of cathode rays is fixed, and q/m >> ions $\Rightarrow$ Electron is discovered

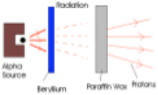
- 1909: Rutherford observes large-angle scattering of alpha particles from a thin gold foil, implying atoms have a compact nucleus with most of atomic mass



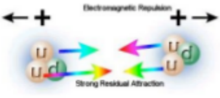
- 1914: Bohr proposes “orbital model” of hydrogen and calculates emission spectrum



- 1932: Chadwick discovers the neutron (solves puzzle of why atomic number and atomic mass differ)

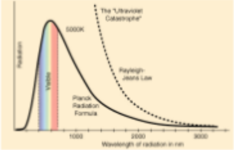


- 1934: Yukawa proposes a theory of the strong force that holds the nucleus together, predicts particle with mass ~$m_p/6$ mediating the strong force

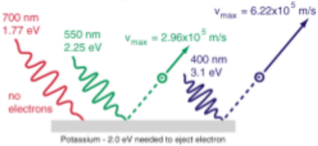


The Carrier of Electromagnetism: γ

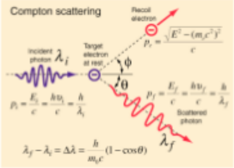
- 1899: Planck proposes that the radiation emitted by a black body is quantized in units of $h\nu$
- 1905: Einstein explains the photoelectric effect using Planck's EM quantum (the photon)
- 1916: Millikan does precision measurements of the photoelectric effect, finds agreement with Einstein's prediction, measures h to 0.5%.
- 1923: Compton observes wavelength shift when light is scattered off electrons, confirming both the existence and the particle nature of photons.



$E_{photon} = h\nu$



Photoelectric effect



Dirac and Antimatter

- 1927: Dirac combines QM and special relativity in an elegant equation.

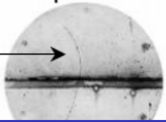
$$H\phi \equiv i\frac{\partial \phi}{\partial t} = (-i\alpha\cdot\nabla + \beta m)\phi$$

We will learn more about this equation later.

Two solutions for the wavefunction ϕ arise unavoidably (since $E^2 = p^2 + m^2$ admits two solutions for the energy, one positive and one negative).

Dirac postulates a “sea” of negative energy electrons. Excitations of the sea would appear as positive energy “holes” of positive charge.

- 1932: Anderson discovers a positron (from a cosmic ray) in a cloud chamber.



Holding the Nucleus Together: The Strong Force

- 1934: Yukawa proposes a theory of the strong force. Like EM and gravity, it employs a field, but in this case it has a massive, rather than a massless, quantum.
- A massive quantum implies a short range ~ $\hbar / m_\pi c$
- In order to agree with known nuclear binding energies, the exchanged quantum (a "meson") would have mass ~$m_p/6$.

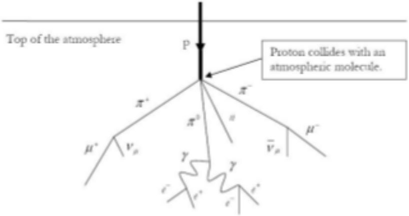
This theory of the strong interaction is not correct; it would be another 30 years before the world of quarks and gluons would be revealed. However, in an appropriate limit (low energies), Yukawa theory is an effective theory of the strong interaction, and the form of the Lagrangian he used recurs in many modern theories.

Mesons

- 1937: A new particle is seen in cosmic rays by two groups. The mass is reasonably close to the expectation for the pion, however,
  - Lifetime was too long?!?
  - Different mass measurements were inconsistent

- 1946: It was seen that these cosmic ray particles (at sea level) appeared to interact weakly with nuclei (pion should interact strongly)

- 1947: Powell looks at cosmic rays at high altitude and shows that there are two mesons: π and μ
  - π is the Yukawa particle, is short-lived (10^{-8} s), and decays to μ (plus a ν)
  - μ lives for 10^{-6} s, does not interact strongly, decays to e (plus two ν)



Beta Decay, Neutrinos

- Radioactivity sometimes leads to emission of e^- (and in a few cases e^+)
- The energy of the electron is less than expected for the two-body decay A $\rightarrow$ B + e:

$$m_A c^2 = \sqrt{m_B^2 c^4 + \left(E_e^2 - m_e^2\right)} + E_e$$

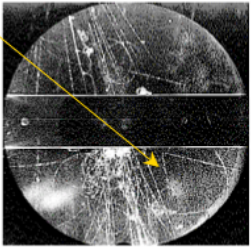
$$E_e = \frac{m_A^2 - m_B^2 + m_e^2}{2m_A} c^2$$

$p_B = p_e$ for a 2-body decay

- 1930: Pauli proposes a missing neutral particle
- 1933: Fermi proposes theory of beta decay involving a "neutrino"
- Neutrinos are also (not) seen in the decay in the decays of charged pions (one missing) and in muons (two missing)
- 1962: neutrinos from pion decay, incident on nuclei, produce μ, not e: ν_μ and ν_e are separate particles.

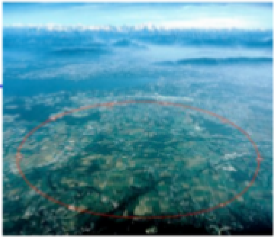
Strange Particles

- 1947: Rochester and Butler discover $K^0 \to \pi^+\pi^-$ in a cloud chamber picture of a cosmic ray.



- 1949: Powell discovers $K^+ \to \pi^+\pi^-\pi^+$
- 1950: Anderson discovers $\Lambda \to p\pi^-$
- 1953: Brookhaven Cosmotron and Berkeley Bevatron make strange particles in the laboratory.
- Some mysteries:
  - Produced strongly but decay slowly
  - Two particles of equal mass have opposite parity; one decays to two pions, the other to three pions…
- New quantum number -- strangeness -- assigned to these new particles; conserved by strong & EM forces, but not by weak
- Amazing phenomenology -- we'll study the K system later on

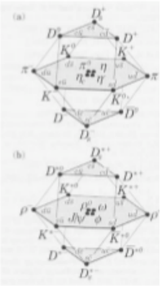
Manmade Accelerators



- 1929: Cyclotron invented by E.O. Lawrence
- 1953: Brookhaven Cosmotron reaches 3.3 GeV
- 1954: Berkeley Bevatron (6.3 GeV)
- 1960s: AGS (30 GeV) at Brookhaven, PS (28 GeV) at CERN
- 1970s: Fermilab main ring, CERN SPS (300-400 GeV)
- 1970s: 5-30 GeV e^+e^- colliding beam machines (SLAC, DESY, Cornell, Frascati)
- 1980s: p-pbar colliders (CERN SPPS, Fermilab Tevatron)
- 1990s: High-energy e^+e^- colliders (CERN LEP, SLAC SLC)
- **2010-2012 Run I at LHC (7/8TeV pp collider) & Run II at LHC to start at CERN in 2015 (13 TeV pp collider)**

Quark Model

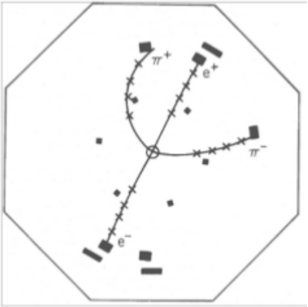
- Many hadrons (mesons and baryons) were discovered by 1964
- Attempts to classify this "zoo" led to the suggestion of quarks (Murray Gell-Mann):
  - 3 types of quarks: up, down, and strange (u, d, s)
  - Carry charges (+2/3, -1/3, -1/3)
  - Appear in combinations of either quark+antiquark, or 3 quarks (or 3 antiquarks)
  - Existing hadrons fit nicely into this scheme



- Scattering e- and ν off nuclei show evidence for substructure, in particular 3 "partons" within each nucleon.
- Why are there no free quarks?
- Pauli exclusion principle should forbid the Ω^- baryon (3 s quarks) but it's seen
  - Quark model is wrong, or
  - There is another hidden quantum number (colour)

Charm, and the "November Revolution"

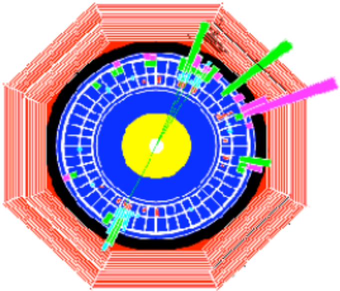
- New massive (3.1 GeV) "J/ψ" particle seen at Brookhaven ($p+N \rightarrow \mu^+\mu^- X$) and SLAC ($e^+e^- \rightarrow$ hadrons) in November 1974
  - Narrow width (i.e. long lifetime) incompatible with existing quark constituents
  - Classified as a bound state of a new quark and antiquark
  - Charm quark is heavy (~1.4 GeV) compared with u, d, and s



- Particles with charm flavor (D mesons: c quark + u or d antiquark) seen subsequently
- Restores symmetry between quarks and leptons: 2 generations with 2 members each (e,ν_e), (μ,ν_μ) and (u,d), (s,c)
- However, symmetry didn't last long…

Tau: a 3rd Charged Lepton

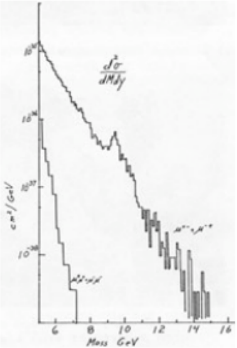
- Shortly after the discovery of the J/ψ some unusual events were seen at SLAC; e.g. $e^+e^- \rightarrow e^+\mu^- +$ missing momentum
- Determined to be from the τ, a heavier (x17) cousin of the μ
- An additional neutrino (ν_τ) was also assumed
- Nature apparently has (at least) 3 generations of leptons

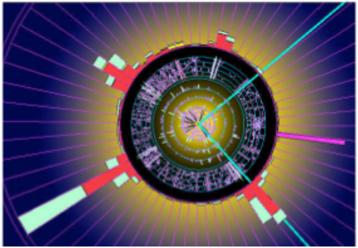


- The discovery suggested that 3rd generation of quarks might exist…

Bottom and Top

- 1977: Lederman et al. find ϒ (upsilon) meson in invariant mass of $\mu^+\mu^-$ pairs; evidence for 5th "b" quark with mass of ~5 GeV
- Implies that at least 3 generations of quarks exist
- 1983: b-flavoured particles seen at Cornell
- Rich phenomenology (and applicability to open questions) makes study of *B* mesons still a forefront activity today.

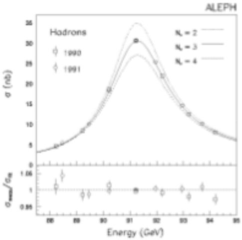




- Top quark is much, much heavier than the others (174 GeV!)
- First seen at Fermilab in 1995, after many years of effort.

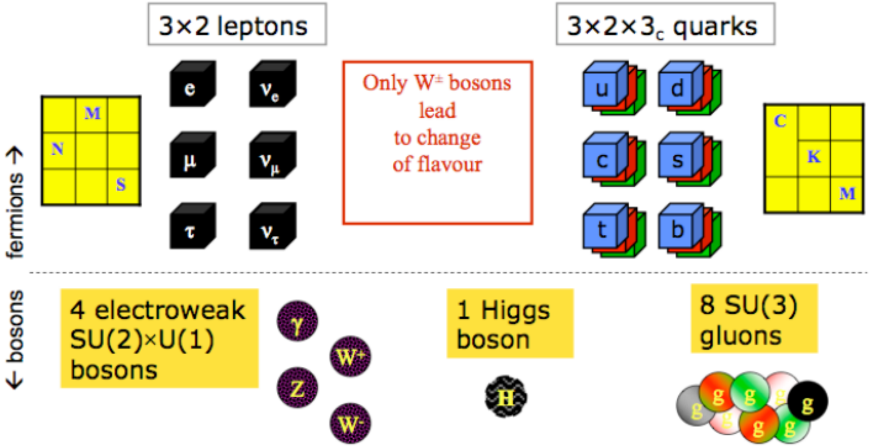
W and Z and ...

- By early 1970s the Standard Model was being formed
- Predicted massive vector bosons as the carriers of the weak force
- Predicted neutral weak currents; seen at CERN in 1973
  - Requires ν-induced interactions to avoid being masked by much stronger EM interactions
- CERN SPPS produces W and Z in 1983, confirming electroweak unification
- LEP collider at CERN and SLC at SLAC produce Z in e^+e^- collisions; detailed studies confirm SM and limit the number of light neutrino species to 3



… the Standard Model

~25 'fundamental' parameters
(masses, couplings, mixings)



Standard Model

The Standard Model - Fermions

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

The Standard Model - Bosons

BOSONS force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

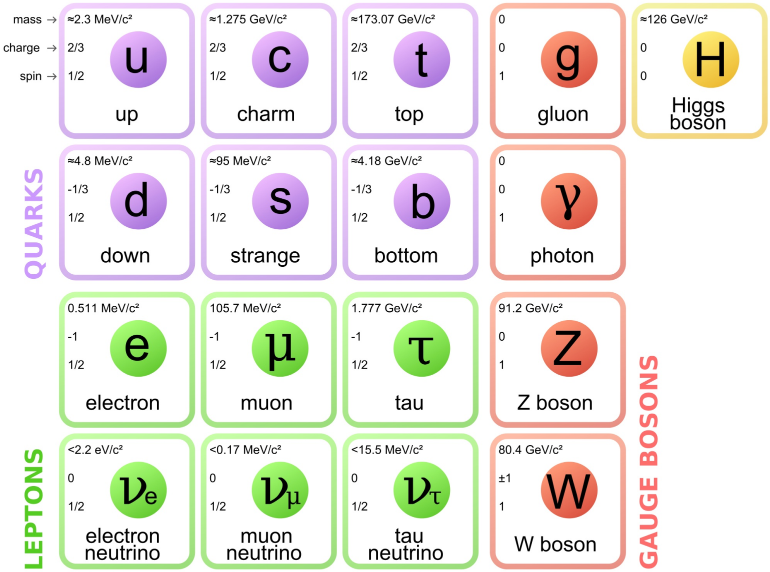
The Standard Model - Interactions

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

The Standard Model

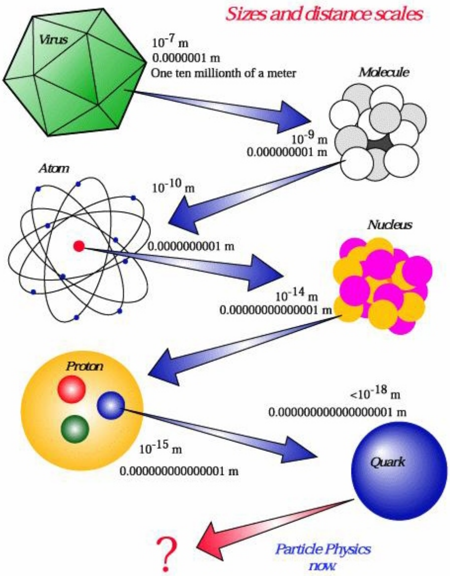


The Standard Model - Interactions

	slow	fast
large	Classical Mechanics	Relativistic Mechanics
small	Quantum Mechanics	Quantum Field Theory

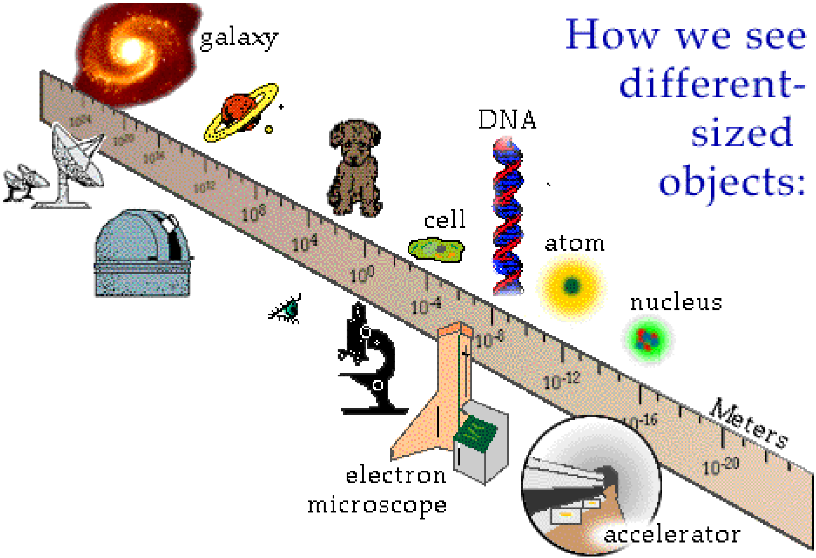
- The theories in the Standard Model are Quantum Field Theories (QFT).
- This is not a QFT course!
- However we can USE the Feynman Rules to calculate measurable quantities.

Particle Physics



- On the way down in scale we have discovered hundreds of particles
- However, the fundamental ones are few...

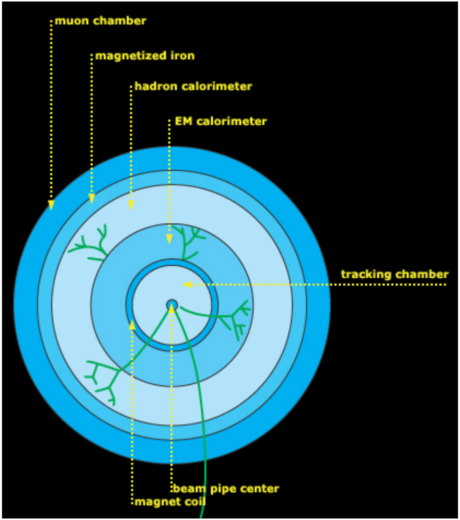
Particle Physics - Tools



We need a short-wavelength probe.

Particle Physics - Detectors

- the probe scatters from the target and must be detected
- the interaction must be reconstructed
- the reconstructed data must be interpreted
- bad models/theories must be discarded



Units in Particle Physics

Energy is measured in MeV, GeV or TeV

$$1\ \text{MeV} = 10^6\ \text{eV},\ 1\ \text{GeV} = 10^9\ \text{eV},\ 1\ \text{TeV} = 10^{12}\ \text{eV}$$

$$1\ \text{TeV} = 1000\ \text{GeV},\ 1\ \text{GeV} = 1000\ \text{MeV} \qquad 1\ \text{eV} = 1.6\ 10^{-19}\text{J}$$

1 eV is the energy gained by one electron subjected to a potential difference of 1 Volt

$$E^2 = p^2c^2 + m^2c^4$$

Where E energy, p momentum, m rest mass. Hence pc and mc^2 have dimensions of energy and it is convenient to measure momentum in units of GeV/c and mass in units of GeV/c^2

$$[E(\text{GeV})]^2 = [p(\text{GeV}/c)]^2c^2 + [m(\text{GeV}/c^2)]^2c^4$$

$$1\ \text{eV}/c^2 = 1.78\ 10^{-36}\text{Kg} \qquad 1\ \text{eV}/c = 5.34\ 10^{-28}\text{Kg ms}^{-1}$$

Because c cancels out we often omit the c i.e. put c = 1 (and $\hbar$ = 1) so momenta and masses are also measured in GeV

c = 3 × 10^8 ms^{-1}
c = 300,000 Km s^{-1}
c = 186,000 mph
c = 1 light year/ year

Other Units

$$\hbar c = 1.05\ 10^{-34}\text{Js} \times 3\ 10^{8}\text{ms}^{-1} \quad \text{(SI units)}$$

$$\hbar c = \frac{1.05\ 10^{-34}\text{Js} \times 3\ 10^{8}\text{ms}^{-1}}{1.6\ 10^{-19}(\text{J/eV})}$$

$$\hbar c = 1.97\ 10^{-7}\text{eV m}$$

$$\hbar c = 1.97\ 10^{-7} \times (10^{-6})\text{MeV} \times (10^{15})\text{fm}$$

$$\hbar c = 197\ \text{MeV fm}$$

$\Delta E \cdot \Delta t = \hbar$ = Energy × time
$\therefore \hbar c$ = Energy × time × velocity
= Energy × distance

Charges measured in term of the electronic charge e

$e = 1.6 \times 10^{-19}$ C

Cross sections measured in terms of barns

1 barn = 10^{-28} m^2

Aside - Wavelength of Probe

Probing small features requires a short-wavelength probe.
We use particles as probes, remember what we learned from de Broglie

$$\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} = \frac{\hbar}{p}$$

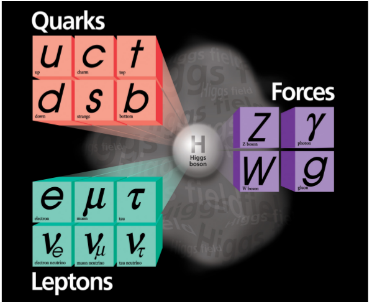
Note: as momentum goes up, wavelength goes down
You can show extreme cases:

$$\begin{aligned}
E^2 &= p^2c^2 + m^2c^4 \\
\lambda\!\!\!^{-} &= \frac{\hbar}{p} = \frac{\hbar c}{\sqrt{E^2 - m^2c^4}} \\
1)\lambda\!\!\!^{-} &= \frac{\hbar c}{\sqrt{2mc^2E_{kin}}}, E_{kin} << m_0c^2 \\
2)\lambda\!\!\!^{-} &= \frac{\hbar c}{E_{kin}}, E_{kin} >> m_0c^2
\end{aligned}$$

A Particle Physicists Periodic Table: The Standard Model

Over the last 100 years: combination of Quantum Mechanics and Special Theory of relativity along with all new particles discovered has led to the Standard Model of Particle Physics. The new (final?) Periodic Table of fundamental elements.

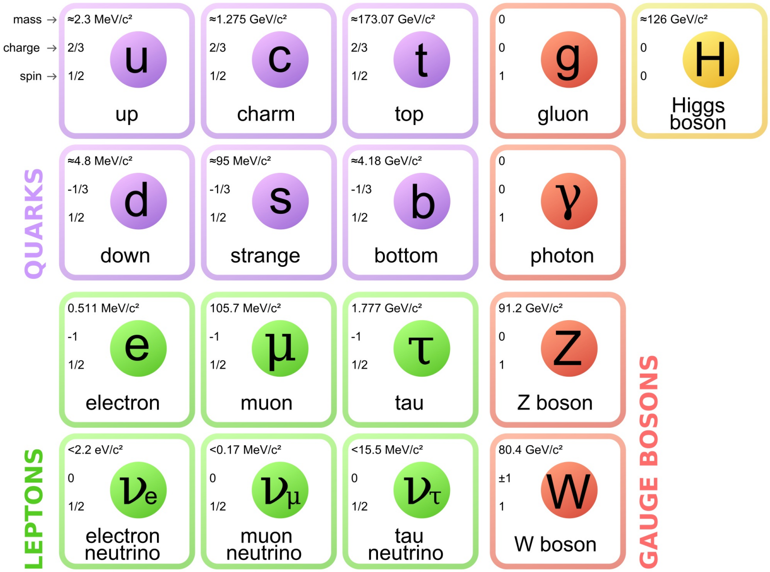
Matter particles



Force particles

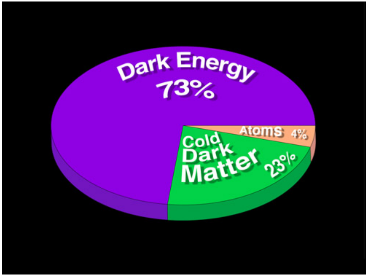
The Standard Model has been tested thousands of times, to excellent precision. Yet, its most basic mechanism, that of granting mass to particles remained a mystery for a long time. A major step forward was made in July 2012 by experiments at Large Hadron Collider with the discovery of a particle that **could be the long-sought Higgs boson!!**

The Standard Model



The Standard Model - Great Mysteries

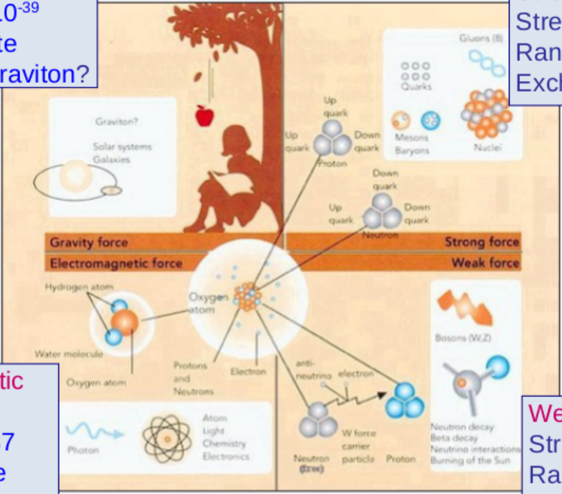
- As brilliant as the SM is, there are problems/mysteries:
  - 20+ free parameters, hierarchy problem, etc.
  - unification of 3 forces?
  - gravity?
  - matter/anti-matter asymmetry?
  - Dark matter candidate?
- So, the ultimate goal is to rule-out the SM and find something better!



Four forces

Gravity
Strength: 6×10^{-39}
Range: Infinite
Exchange: Graviton?

Strong Force
Strength: 1
Range: 10^{-15}m
Exchange: Gluon

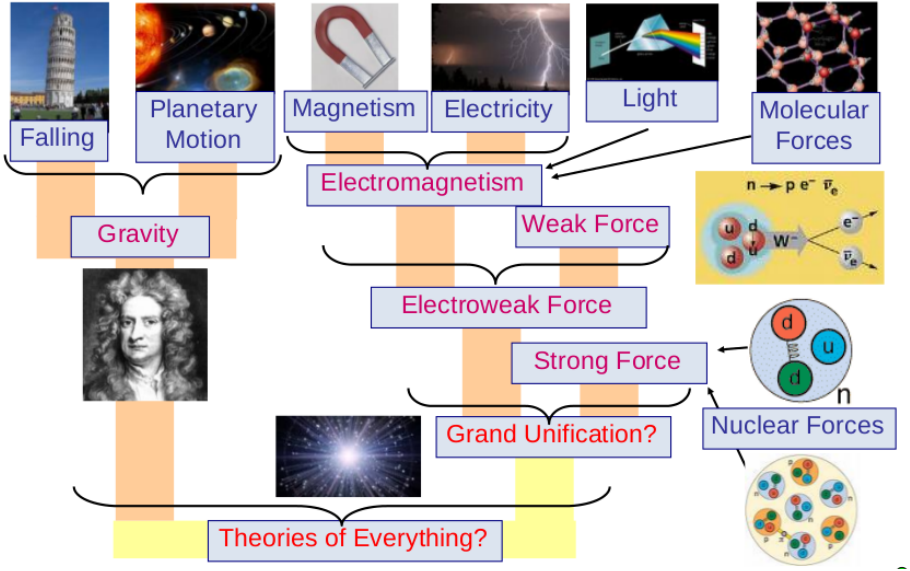


Taken from http://universe-review.ca/F15-particle.htm

Electromagnetic Force
Strength: 1/137
Range: Infinite
Exchange: Photon

Weak Force
Strength: 10^{-6}
Range: 10^{-18}m
Exchange: $W^{\pm} Z^0$

Unification of forces



Forces

The weak force acts between all quarks and leptons

The electromagnetic force acts between all charged particles

The strong force acts between all quarks

	Weak	EM	Strong
Quarks	✓	✓	✓
Charged Leptons	✓	✓	✗
Neutral Leptons	✓	✗	✗

Range of forces

The range of the interaction is related to the mass of the exchange particle M

An amount of energy $\Delta E = Mc^2$ is 'borrowed' for a time Δt governed by the Uncertainty Principle

$$\Delta E \Delta t \sim \hbar \qquad \Delta t = \hbar / \Delta E$$

The maximum distance the exchange particle can travel in this time is:

$$\Delta x = c \Delta t$$

(c is the maximum velocity it can have)

$$\Delta x = c\hbar / \Delta E = c\hbar / Mc^2$$

$$\Delta x = \hbar c / Mc^2$$

ħc in funny units

The photon has zero mass → infinite range

Converts GeV to MeV

The W has a mass of ~80 GeV/c^2 → 197 MeV fm / 80×10^3 MeV → 2×10^{-3} fm

Elementary Particles

Elementary Particles are 'point like' fundamental spin ½ fermions (obey Fermi-Dirac Statistics). They have no discernible size or structure. There are two types Quarks and Leptons:

Quarks – have electric charge -⅓ e or +⅔ e

Quarks come in 6 'flavours' :

up (u), down (d), strange (s), charm (c), bottom (b), top (t)

Each quark comes in 3 'colours' - Red, Green, Blue

Leptons - have electric charge 0 or ±e

Leptons also come in 6 'flavours' :

electron e^-, muon μ^-, tau τ^-,

electron neutrino ν_e, muon neutrino ν_μ, tau neutrino ν_τ

Leptons don't have colour

Antiparticles

All quarks and leptons have antiparticle partners, with all quantum numbers reversed, but with the same masses

Antiquarks: $\bar{d}$, $\bar{u}$, $\bar{s}$, $\bar{c}$, $\bar{b}$, $\bar{t}$

Put a bar over the symbol

Charge + rather than -

Antileptons: e^+, μ^+, τ^+, $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$

The anti-electron is known as the positron

Antiquarks have anticolours

Note: Electric Charge has values +, - , 0

Colour Charge has values r, g, b, $\bar{r}$, $\bar{g}$, $\bar{b}$

When particle and antiparticle meet they annihilate to give energy with all quantum numbers zero (usually as photons)

Aside: Fermions & Bosons

Consider 2 particles 1 and 2 in two states α and β. There are 2 linear combinations to form and overall wavefunction

$$\psi_S = \frac{1}{\sqrt{2}}(\psi_1(\alpha)\psi_2(\beta) + \psi_1(\beta)\psi_2(\alpha))$$
$$\psi_A = \frac{1}{\sqrt{2}}(\psi_1(\alpha)\psi_2(\beta) - \psi_1(\beta)\psi_2(\alpha))$$

If we interchange particles $1 \to 2$, $2 \to 1$

$$\psi_S \to \frac{1}{\sqrt{2}}(\psi_2(\alpha)\psi_1(\beta) + \psi_2(\beta)\psi_1(\alpha)) = \psi_S$$

Symmetric

$$\psi_A \to \frac{1}{\sqrt{2}}(\psi_2(\alpha)\psi_1(\beta) - \psi_2(\beta)\psi_1(\alpha)) = -\psi_A$$

Antisymmetric

If the two particles are in the same state i.e. $\alpha = \beta$

$$\psi_S = \frac{1}{\sqrt{2}} 2\psi_1\psi_2 \qquad \psi_A = 0$$

Two particles with antisymmetric wavefuntions cannot be in the same state $\to$ Pauli Exclusion Principle ($\to$ Shell structure of electrons)

Aside: Fermions & Bosons

Particles with antisymmetric wavefunctions are called Fermions and obey Fermi-Dirac statistics

e.g. leptons, quarks and baryons (electrons, proton, neutron . . .)

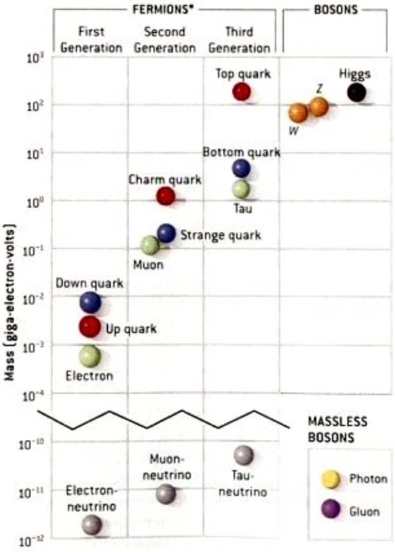
All fermions have half integer spins $\frac{1}{2}$, $\frac{3}{2}$ . . .

Particles with symmetric wavefunctions are called Bosons and obey Bose-Einstein statistics

e.g. exchange particles and mesons (γ, W, Z, gluon, π, K . ..)

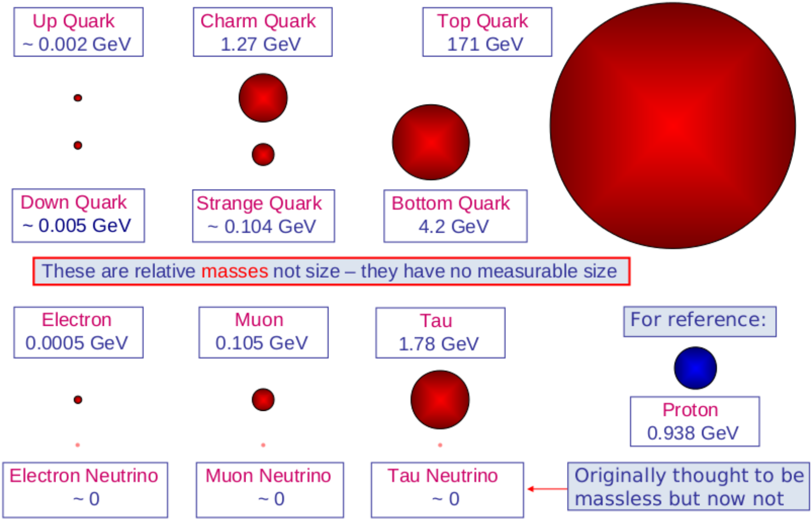
All bosons have integer spins 0, 1, 2 . . .

Particle Masses

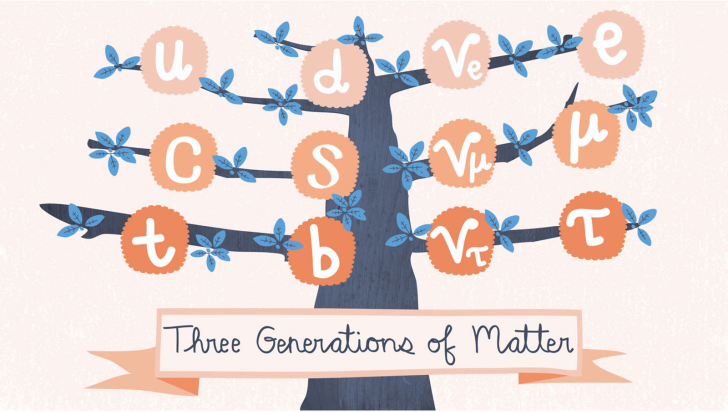


Particle Masses

Quarks and Leptons can be arranged in 3 'families' or 'generations':



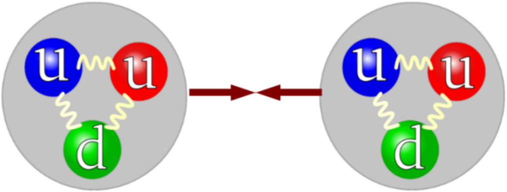
Why three generations?



The mystery of particle generations on Symmetry Magazine

Hadron Colliders

A hadron collider is a **broadband** quark and gluon collider.



Hadron:

In particle physics, a hadron is a composite particle made of quarks held together by the strong force. Hadrons are categorized into two families:

- baryons, such as protons and neutrons, made of three quarks and
- mesons, such as pions, made of one quark and one antiquark.

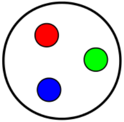
Hadrons

Baryons consist of 3 quarks (Antibaryons 3 antiquarks)

e.g. proton uud (antiproton $\bar{u}\bar{u}\bar{d}$) $\frac{2}{3}e + \frac{2}{3}e + (-\frac{1}{3}e) = e$

neutron udd (antineutron $\bar{u}\bar{d}\bar{d}$) $\frac{2}{3}e + (-\frac{1}{3}e) + (-\frac{1}{3}e) = 0$

proton

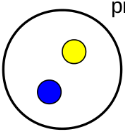


Mesons (antimesons) consist of a quark and an antiquark

e.g. pion π^+ $u\bar{d}$ (antipion π^- $\bar{u}d$) $\frac{2}{3}e + \frac{1}{3}e = e$

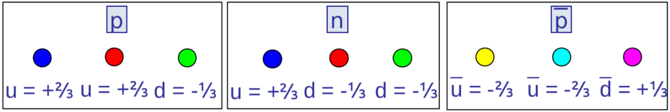
neutral pion π^0 $u\bar{u}$ or $d\bar{d}$ $\frac{2}{3}e + (-\frac{2}{3}e) = 0$

pion

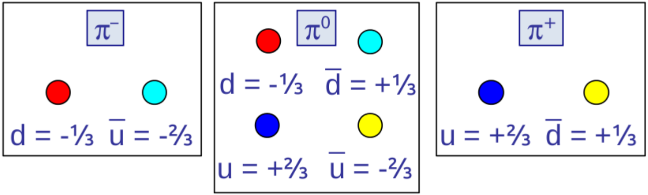


Common Hadrons

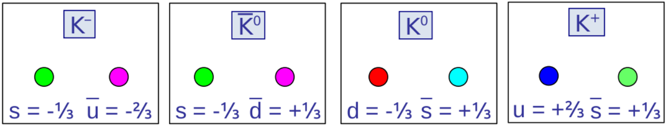
Nucleons



Pions



Kaons

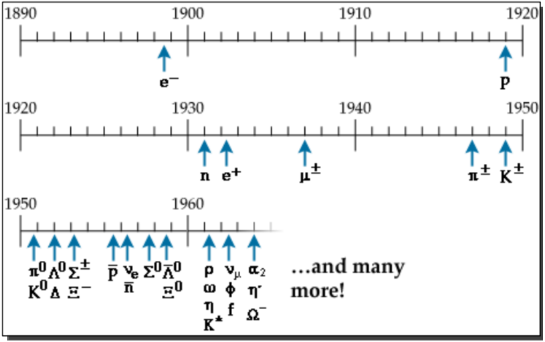


Particle Physics - The Zoo

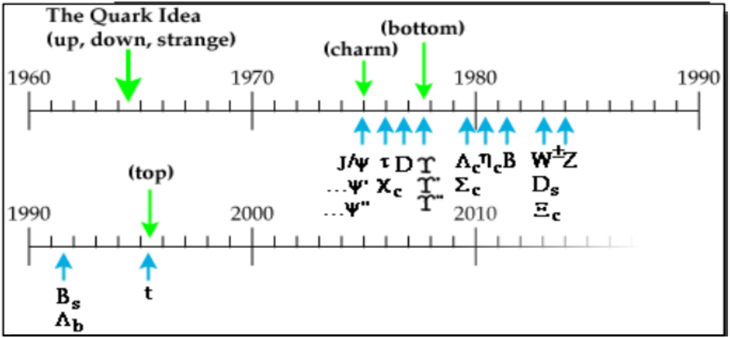
- For a brief, happy, period (1930s) we had it all. The fundamental particles were: proton, neutron, electron (OK, photons too).
- electrons are so-called leptons (light-weights), protons and neutrons are baryons (heavy-weights)
- However, now there are (VERY incomplete list)
  - Mesons: $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, K^0_S, K^0_L, \eta, \eta', \rho, \phi, D, D_s, J/\psi, B, B_s, B_c, \upsilon$
  - Baryons: $p, n, \Delta^{++}, \Delta^+, \Delta^0, \Delta^-, \Lambda^0, etc., \Sigma, \Xi, \Omega$

Particle Physics - The Zoo

All these particles explained by combinations of more fundamental 'quarks', u, d, s and their anti-quarks



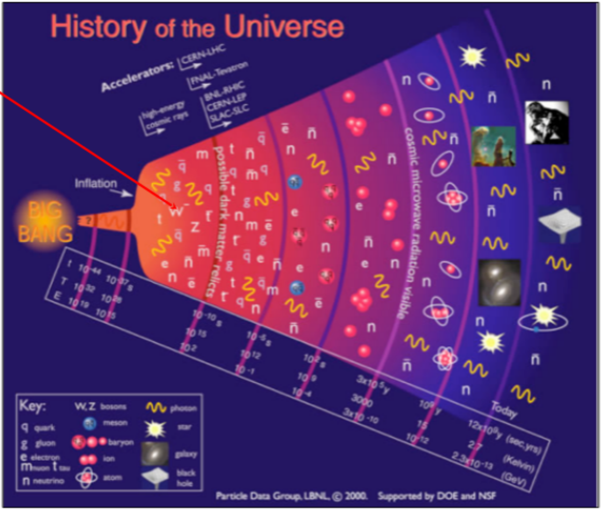
Particle Physics - Six Quarks



History of Universe

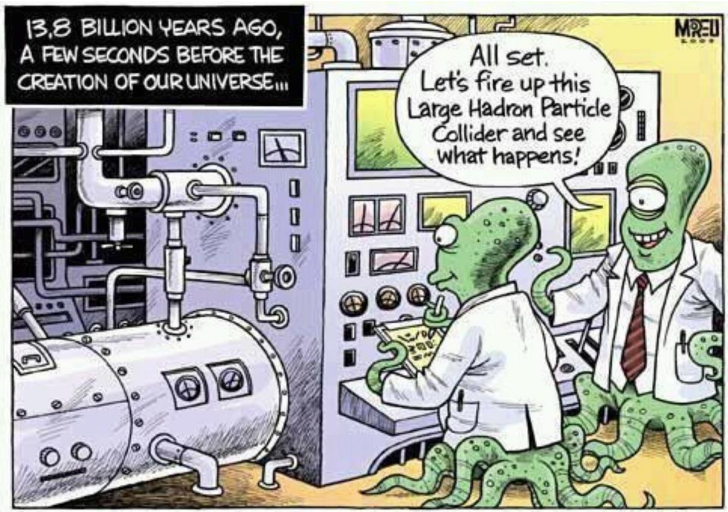
Particle Physics

Recreating the conditions just after the Big Bang



Astronomy

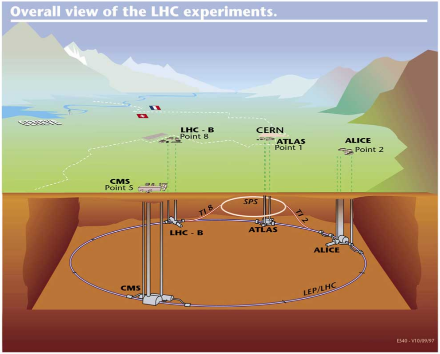
Large Hadron Collider



Study physics laws of first moments after Big Bang

Large Hadron Collider

The Large Hadron Collider (LHC) is the worlds largest and most powerful particle accelerator.



Fun Facts

- 27km circumference, 100m under ground
- 9300 magnets, worlds largest fridge
- Magnets colder and beam pipes emptier than space
- 100,000 times hotter than the heart of the sun
- Protons at 99.9999991% of the speed of light

Particle Physics and Cosmology

Increasing connection between Particle Physics and Cosmology – the more fundamental the particle, the earlier they were created during the early Universe (Big Bang)

Today's particle accelerators probe further and further back in time to moment of Big Bang:

Energy ≡ Temperature

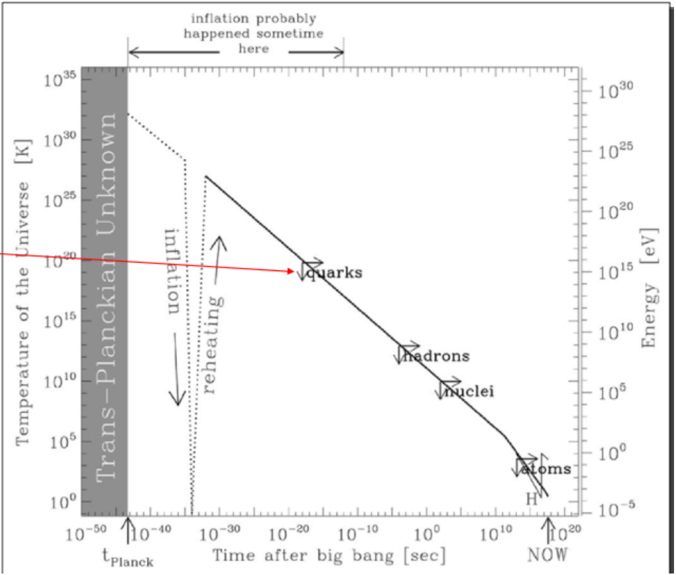
$$E \approx kT$$

where k is Boltzmann's constant ($8.6\ 10^{-5}$ eV K^{-1})

At LHC Accelerator energy ~ 14 TeV
$\equiv 14\times10^{12}$ eV / $8.6\ 10^{-5}$ eV $K^{-1} \equiv 1.6\ 10^{17}$ K
Equivalent to $< 10^{-14}$ s after the Big Bang

Probing the Early Universe

Particle Physics today probes this region



Nuclear Physics

All the slides are available on google Drive link:

`https://goo.gl/vepcJi`

goo.gl/vepcJi

List of students registered on AMS is available at google's spreadsheet link:

`https://goo.gl/pm7Xaa`

goo.gl/pm7Xaa

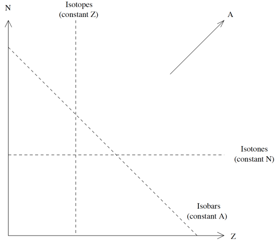
Please provide your e-mail ID and 8 or 10 point system

Nuclei

The atomic number. The atomic number Z gives the number of protons in the nucleus. The charge of the nucleus is, therefore, $Q = Ze$, the elementary charge being $e = 1.6 \cdot 10^{-19}$ C. In a neutral atom, there are Z electrons, which balance the charge of the nucleus, in the electron cloud. The atomic number of a given nucleus determines its chemical properties.

The mass number. In addition to the Z protons, N neutrons are found in the nucleus. The mass number A gives the number of nucleons in the nucleus, where $A = Z + N$. Different combinations of Z and N (or Z and A) are called *nuclides*.

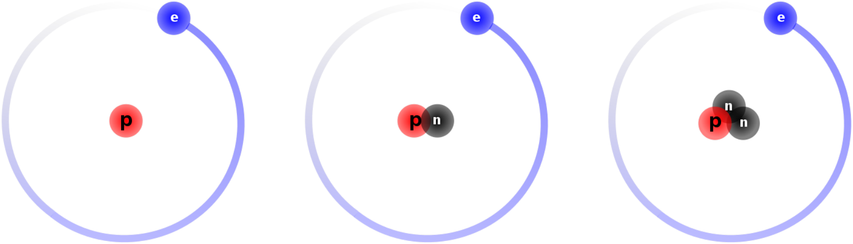
– Nuclides with the same mass number A are called *isobars*.
– Nuclides with the same atomic number Z are called *isotopes*.
– Nuclides with the same neutron number N are called *isotones*.



A nucleus is described by the number of protons, Z, and the number of neutrons, N. The total number of protons and neutrons (collectively called nucleons) is denoted by the atomic mass number A which is by definition $Z + N$.

Typically a element X is represented as: ${}^{A}_{Z}X$ or ${}^{A}X$ or ${}^{A}_{Z}X_{N}$
Ex: Hydrogen ${}^{1}_{1}H$ or ${}^{1}H$ or ${}^{1}_{1}H_{0}$

Schematic of Istotopes of Hydrogen



$^{1}_{1}H$ $^{2}_{1}H$ $^{3}_{1}H$

Protium **Deuterium** **Tritium**

Abundances of the elements

- Why are we interested in the abundances of the elements?
  $\rightarrow$ Constitution of (baryonic) matter, quantities of stable elements/isotopes
- Formation of the solar system
  $\rightarrow$ Composition of the solar system, planetary compositions
- Origin of the elements
  $\rightarrow$ Abundance distributions are critical tests for nucleosynthesis models

Nucleosynthesis

- Nucleosynthesis is the process that creates new atomic nuclei from pre-existing nucleons (via **nuclear fusion**), primarily protons and neutrons.
- The Sun works by combining protons into heavier nuclei, so giving out **fusion energy**. In fact, almost all nuclei heavier than helium were made in stars.
- We have to understand why the Sun is powered by hydrogen in the first place.

Big Bang Nucleosynthesis - 1

In the very early Universe, specifically within the first 4 minutes after the Big Bang, matter was too hot to form nuclei; the collisions were just too energetic for any nuclear binding to survive. The matter was all separate particles, namely electrons, neutrinos, protons and neutrons; the latter have a lifetime of around 15 minutes and so had not decayed significantly yet. There were also a lot of photons up to high energies. However, as the Universe expanded and hence cooled, it reached a temperature of around 10^9 K which corresponds to 0.1 MeV, when larger nuclei could form with a good probability of survival.

The first reaction which could occur was

$$p + n \to d + \gamma$$

which has $Q = 2.2$ MeV. This low value indicates the deuteron is not strongly bound and it is the ease of disintegrating this initial bound state which prevented nuclei from forming earlier. However, after 4 minutes, enough of the deuterons can live for a significant time that further reactions with the nucleons can occur

$$p + d \to {}^3_2\text{He} + \gamma, \qquad n + d \to {}^3_1\text{H} + \gamma$$

followed by

$$p + {}^3_1\text{H} \to {}^4_2\text{He} + \gamma, \qquad n + {}^3_2\text{He} \to {}^4_2\text{He} + \gamma$$

The helium nuclei are strongly bound and so are not easily knocked apart again; hence, once past the deuteron bottleneck, these reactions proceed quickly and give mainly helium.

However, to get further fusion after ${}^4_2$He is hard. There are no $A = 5$ nuclei so protons and neutrons could not fuse with the ${}^4_2$He. Also, the density of the other nuclei was too low for many of the deuterons, tritium or ${}^3_2$He to react. Even if reactions such as

$$d + {}^4_2\mathrm{He} \to {}^6_3\mathrm{Li} + \gamma, \qquad t + {}^4_2\mathrm{He} \to {}^7_3\mathrm{Li} + \gamma$$

occured, the lithium can be easily destroyed by

$$n + {}^6_3\mathrm{Li} \to {}^4_2\mathrm{He} + t, \qquad p + {}^7_3\mathrm{Li} \to 2 \times {}^4_2\mathrm{He}$$

The probability of three ${}^4_2$He reacting together to form ^{12}C, the next nucleus more strongly bound than helium, is negligible also. Hence most of the nuclear matter in the Universe was still single nucleons, with some helium in addition.

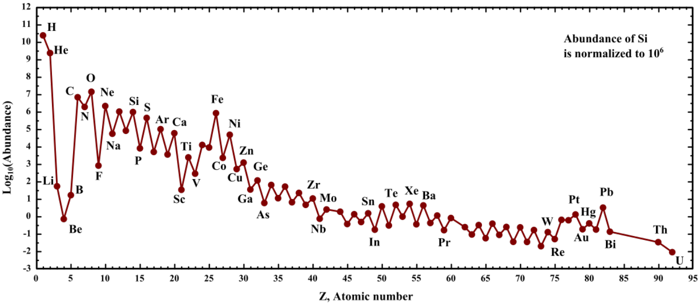
Big Bang Nucleosynthesis - 3

After about 30 minutes, the temperature had dropped by around a factor of three and most of the neutrons has decayed. The latter shut off the deuteron creation reaction and the lower temperature made the other fusions much less likely due to the Coulomb barrier. Hence, the

nuclei were "frozen" after 30 minutes and remained in that state for millions of years until stars formed. The composition of the Universe during this time was (by mass) 76% protons, 24% helium and traces of deuterons, ${}^{3}_{2}\mathrm{He}$ and ${}^{7}_{3}\mathrm{Li}$, as all the other species, including the neutrons and tritium, had decayed.

Even since their creation, the stars have only fused a small percentage of the matter in the Universe so the overall composition of the Universe today is close to how it was 30 minutes after the Big Bang.

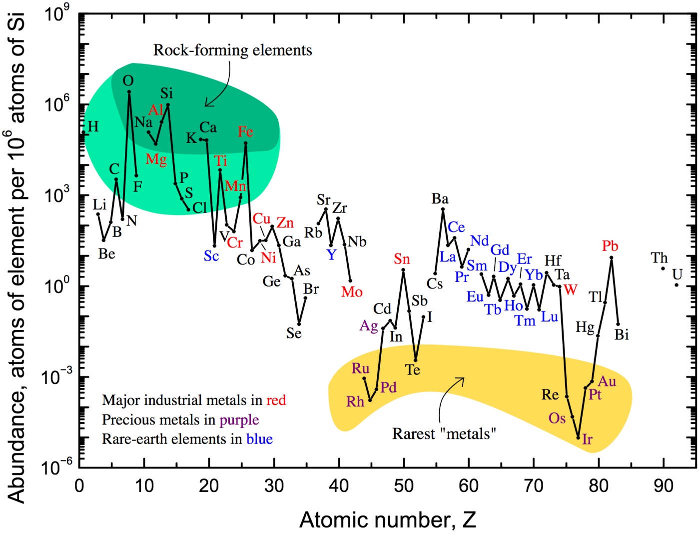
Adundance: Solar System



General Trend: alternation of abundance in elements for even and odd Z & adundance decreases as Z increases

According to current thinking, the synthesis of the presently existing deuterium and helium from hydrogen fusion mainly took place at the beginning of the universe (minutes after the big bang). Nuclei up to ^{56}Fe, the most stable nucleus, were produced by nuclear fusion in stars. Nuclei heavier than this last were created in the explosion of very heavy stars (supernovae)

Adundance: Earth crystal elements



Sun & Earth

From Big Bang Nucleosynthesis (BBN) (covered), Stellar Nucleosynthesis (not covered) & formation of Heavy Nuclei (not covered) the conclusion is that:

Spectroscopy of the Sun shows it contains small amounts of carbon, oxygen and higher nuclei, which could not have been manufactured by the Sun itself, given its temperature. In addition, the Earth contains both thorium and uranium heavy nuclei which have $A > 209$. The only conclusion is that the Earth (and also the Sun) must be the product of the remnants of a supernova which exploded many billions of years ago. All the matter we see on Earth, including ourselves, was formed in the centre of at least one star before our Sun even existed. This material was blasted out into space when the star exploded at the end of its lifetime. We owe our existance to the recycling of matter through the stars.

Odd-Even distribution of the elements as a function of atomic number is also seen for heavy elements

Good example: rare earth elements (REE)
Similar geochemical behavior

Odd-even abundance effect not (completely) erased during re-distribution of REE among minerals in rocks

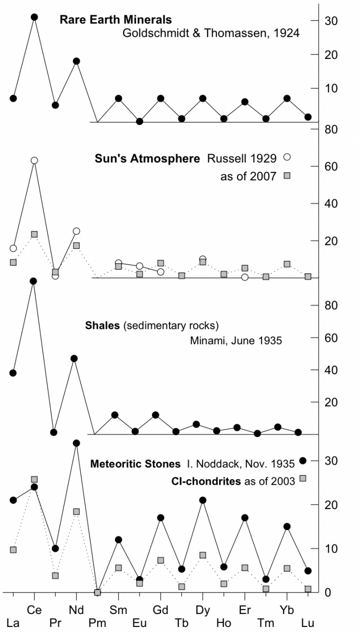


Figure updated from Goldschmidt 1937
Normalized to Y= 100 atoms (Y not shown)

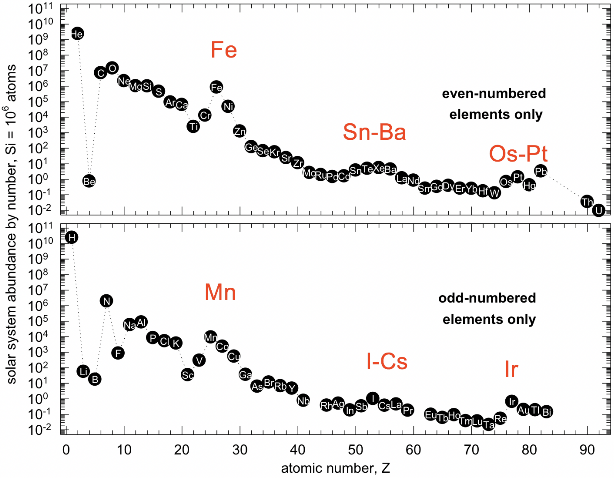
Elemental abundances as function of atomic number (= proton number=Z) show "peaks" e.g., O (N), Fe (Ni), Ba (I), Os (Ir)

Do not follow electron shell stabilities
(noble gases are not the most abundant heavy elements)

280 naturally occurring stable (266) and long-lived nuclides (14)

Nuclides belonging to the same element (same proton number Z) are *isotopes*
"*isos* + *topos*" = in the same place in the periodic table

Separate for even and odd-numbered elements



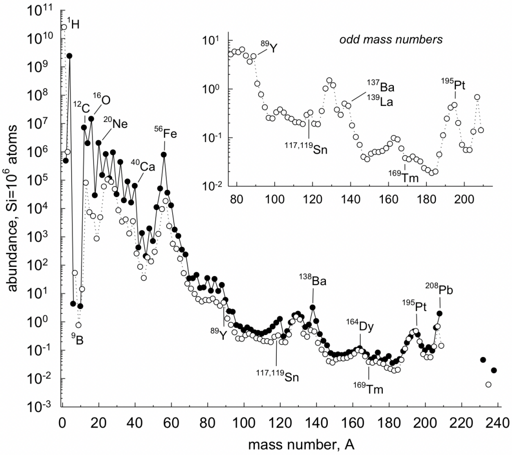
Look at nuclide distributions to decipher what controls elemental abundances

Abundances of the nuclides versus mass numbers

Abundances peak at mass numbers for closed proton and neutron shells

"magic numbers"
2, 8, 20, 28, 50, 82, 126

Doubly-magic nuclei; e.g.
^{4}He Z = N = 2
^{16}O Z = N = 8
^{40}Ca Z = N = 20



<u>266 stable nuclei</u>
Z even, N even: 159 nuclides
Z even, N odd: 53 nuclides
Z odd, N even: 50 nuclides
Z odd, N odd: 4 nuclides (^{2}H, ^{6}Li, ^{10}B, ^{14}N)

Lower number of odd-Z numbered isotopes
→lower abundances of odd-Z numbered elements

Properties of Nuclei

Static Properties

- Electric charge
- Radius
- Mass
- Binding energy
- Angular momentum
- Parity
- Magnetic dipole moment
- Electric quadrupole moment
- Energies of excited states

Dynamic Properties

- Decay and reaction probabilities
- scattering cross sections

Radius of nucleus

The nuclear radius (R) is considered to be one of the basic quantities that any nuclear model must predict.
For stable nuclei the nuclear radius is roughly proportional to the cube root of the mass number (A) of the nucleus, and particularly in nuclei containing many nucleons, as they arrange in more spherical configurations.

The stable nucleus has approximately a constant density and therefore the nuclear radius R can be approximated by the following formula:
$R = R_0 A^{1/3}$
where $R_0 = 1.2 fm$, experimentally determined constant.

The nuclear force has a very short range $\sim$ 1 fm, which is only the size of a nucleon radius. It also saturates, i.e. becomes very large and repulsive for short distances. This means that in a nucleus with many nucleons, they will not all crowd together at the origin but will spread out to occupy a finite volume each, liked packed spheres.

Binding Energy

The binding energy of a system gives information about its binding and stability. This energy is the difference between the mass of a system and the sum of the masses of its constituents.
For nuclei this difference is close to 1% of the nuclear mass. This phenomenon, historically called the mass defect, was one of the first experimental proofs of the massenergy relation $E = mc^2$. The mass defect is of fundamental importance in the study of strongly interacting bound systems.
For stable nuclei, Binding Energy is > 0

The binding energy B is usually determined from atomic masses [AM93], since they can be measured to a considerably higher precision than nuclear masses. We have:

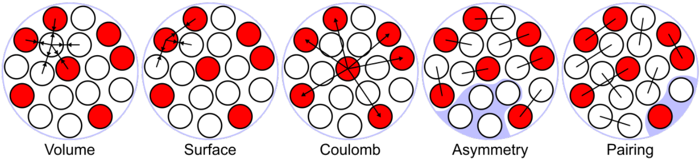
$$B(Z,A) = \left[ZM(^1\mathrm{H}) + (A-Z)M_\mathrm{n} - M(A,Z)\right] \cdot c^2 \,. \tag{2.2}$$

Here, $M(^1\mathrm{H}) = M_\mathrm{p} + m_\mathrm{e}$ is the mass of the hydrogen atom (the 13.6 eV binding energy of the H-atom is negligible), M_n is the mass of the neutron and $M(A,Z)$ is the mass of an atom with Z electrons whose nucleus contains A nucleons. The rest masses of these particles are:

$$\begin{aligned} M_\mathrm{p} &= 938.272\ \mathrm{MeV}/c^2 = 1836.149\ m_\mathrm{e} \\ M_\mathrm{n} &= 939.566\ \mathrm{MeV}/c^2 = 1838.679\ m_\mathrm{e} \\ m_\mathrm{e} &= 0.511\ \mathrm{MeV}/c^2. \end{aligned}$$

Liquid drop model or Semi-empirical mass formula

The terms in the semi-empirical mass formula, which can be used to approximate the binding energy of many nuclei, are considered as the sum of five types of energies. Then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus:



- **Volume energy:** When an assembly of nucleons of the same size is packed together into the smallest volume, each interior nucleon has a certain number of other nucleons in contact with it. So, this nuclear energy is proportional to the volume.

- **Surface energy:** A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. This surface energy term takes that into account and is therefore negative and is proportional to the surface area.
- **Coulomb Energy:** The electric repulsion between each pair of protons in a nucleus contributes toward decreasing its binding energy.
- **Asymmetry energy (also called Pauli Energy):** An energy associated with the Pauli exclusion principle. Were it not for the Coulomb energy, the most stable form of nuclear matter would have the same number of neutrons as protons, since unequal numbers of neutrons and protons imply filling higher energy levels for one type of particle, while leaving lower energy levels vacant for the other type.
- **Pairing energy:** An energy which is a correction term that arises from the tendency of proton pairs and neutron pairs to occur. An even number of particles is more stable than an odd number.

Volume and Surface terms

- The volume of the nucleus would be expected to increase linearly with the number of nucleons, so $V \propto A = N + Z$. Since $V = 4\pi R^3/3$, the nuclear radius R should go as $R \propto A^{1/3}$. The second consequence is that binding energy for each nucleon would be expected to be constant. This is because each nucleon only binds to its nearest neighbours, so the contribution to the binding energy from each is a fixed value. Hence, we would expect $B_E \propto A$, which is again found to be roughly true; the binding energy for heavy nuclei is found to be roughly 8 MeV per nucleon.
- For the nucleons on the surface of the nucleus have as many nearest neighbour bonds as the ones well within the nuclear volume as there are no nearest neighbours outside the nucleus. The correction to the binding energy proportional to the sphere surface area $4\pi R^2$

$\longrightarrow B_E = a_v A - a_s A^{2/3}$

Coulomb term

- There is an EM repulsive force between protons due to their charge and so this will reduce the binding energy for nucleons with several protons. As we believe the nuclear force itself is independent of nucleon type, then the protons will on average be spread evenly throughout the nucleus, which means the charge density is uniform.

$$\Delta B_E = -\frac{3}{5}\frac{e^2}{4\pi\epsilon_0 r_0}\frac{Z^2}{A^{1/3}} = -a_c\frac{Z^2}{A^{1/3}}$$

The above equation says even one proton, i.e. $Z = 1$, gives a correction to the binding energy, even though there is nothing to repel it. Correction of $Z(Z-1)$ is added.

$\longrightarrow B_E = a_v A - a_s A^{2/3} - a_c Z(Z-1)/A^{1/3}$

Asymmetry term

- As long as mass numbers are small, nuclei tend to have the same number of protons and neutrons. Heavier nuclei accumulate more and more neutrons, to partly compensate for the increasing Coulomb repulsion by increasing the nuclear force. This creates an asymmetry in the number of neutrons and protons. The dependence of the nuclear force on the surplus of neutrons is described by the asymmetry term $(N - Z)^2/A$

$$\longrightarrow B_E = a_v A - a_s A^{2/3} - a_c Z(Z-1)/A^{1/3} - a_a (N-Z)^2/A$$

Pairing term

1. Even-even, meaning an even number of both protons and neutrons, and hence even A. This has both pairs strongly bound.
2. Odd-odd, meaning an odd number of both protons and neutrons, and hence also even A. This is the least strongly bound.
3. Even-odd, meaning an even number of one type and an odd number of the other, and hence odd A. This has one strongly bound pair and so should be half way in between the previous two.

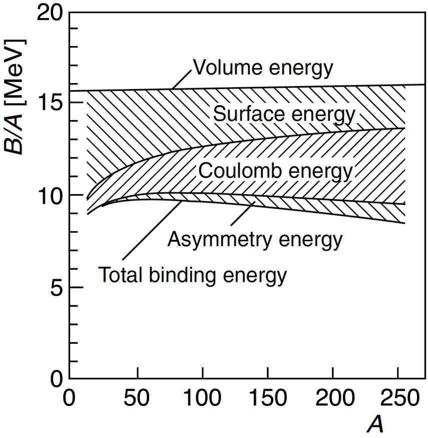
Pairing term: $a_p/A^{1/2}$
a_p takes positive value for even-even nuclei
negative for odd-odd nuclei and
zero for even-odd ones

Binding Energy: Semi-empirical Mass formula

$$B_E = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{1}{A^{1/2}}$$

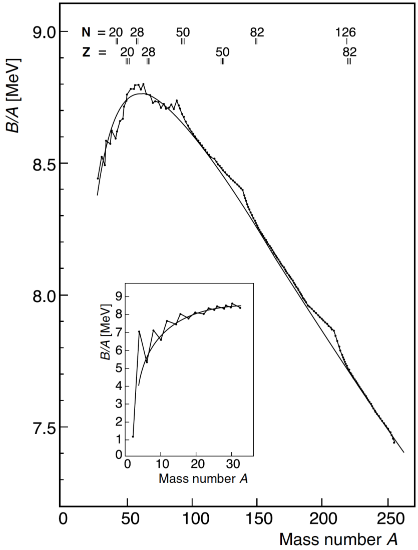
and this is the semi-empirical mass formula. The best-fit parameters take values 15.8 MeV, $a_s = 18.3$ MeV, $a_c = 0.71$ MeV, $a_a = 23.2$ MeV and $a_p = \pm 11.2$ MeV.

Binding Energy: Contributions from various terms



The different contributions to the binding energy per nucleon versus mass number A. The horizontal line at $\approx$ 16 MeV represents the contribution of the volume energy. This is reduced by the surface energy, the asymmetry energy and the Coulomb energy to the effective binding energy of $\approx$ 8 MeV (lower line).

Binding Energy per Nucleon



Binding energy per nucleon of nuclei with even mass number A. The solid line corresponds to the Weizsacker mass formula. Nuclei with a small number of nucleons display relatively large deviations from the general trend, and should be considered on an individual basis. For heavy nuclei deviations in the form of a somewhat stronger binding per nucleon are also observed for certain proton and neutron numbers (Magic Numbers).

Nuclear Stability or β- Stability curve

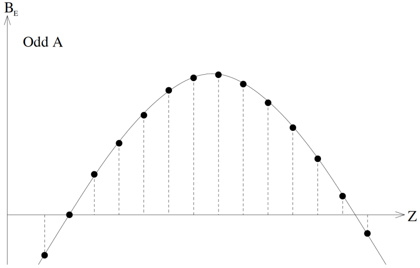
The semi-empirical mass formula is a function of two variables, as $A = N + Z$. It gives the binding energy of the ground state of any nucleus, i.e. any values of Z and N.

$$B_E = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{1}{A^{1/2}}$$

We have not observed nuclei with most of the combinations of Z and N which might be thought possible in the whole of the Z, N plane because the majority of them are highly unstable. The binding energy is largest in a specific region of the Z, N plane which is called the beta-stability valley ("valley" as the mass is at a minimum).
It is easiest to analyse the binding energy along lines of constant A and we will look at it as a function of Z. Putting $N = A - Z$, then

$$B_E = \left(a_v A - a_s A^{2/3} - a_a A + \frac{a_p}{A^{1/2}}\right) + \left(\frac{a_c}{A^{1/3}} + 4a_a\right) Z + \left(-\frac{a_c}{A^{1/3}} - \frac{4a_a}{A}\right) Z^2$$

Odd A

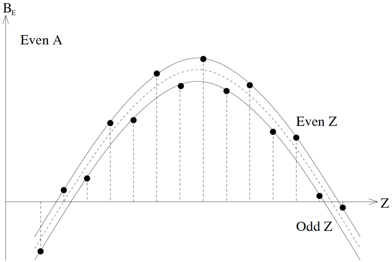


There is a particular value of Z for which the binding energy is maximum, but away from this value, the binding energy falls and eventually goes negative at which point the nucleus is no longer bound.

Even within the region of positive binding energy, the non-maximum values of Z are not necessarily stable; as we will see, beta decay allows them to change protons to neutrons and vice versa, and so move along the curve to the maximum value.

Hence, we only see the reasonably long-lived nuclei which are at or near the maximum.

Even A



For even A, then as Z changes by one, then Z (and N, given that A is fixed and even) goes from even to odd or vice versa.

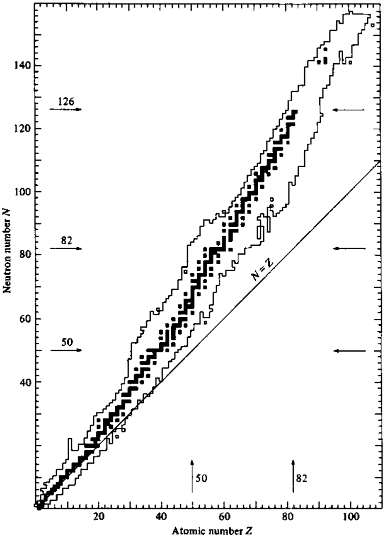
This means the pairing term changes sign. Hence, this shifts the quadratic curve up and down by $\pm a_p / A^{1/2}$ for the alternating even and odd Z values.

However, the same stability arguments still hold; only nuclei near the peak live long enough to be seen.

$$B_E = \left(a_v A - a_s A^{2/3} - a_a A + \frac{a_p}{A^{1/2}}\right) + \left(\frac{a_c}{A^{1/3}} + 4a_a\right) Z + \left(-\frac{a_c}{A^{1/3}} - \frac{4a_a}{A}\right) Z^2$$

The two terms driving BE as a function of Z are the Coulomb and asymmetry terms; only a_c and a_a appear in the Z and Z^2 terms.
If $a_c = 0$, then clearly $N = Z = A/2$ gives the maximum binding energy as the nuclei like to have equal numbers of levels filled.
Conversely, if $a_a = 0$, then $Z = 0$ or 1 to minimise the Coloumb term and hence maximise the binding energy by limiting the amount of Coulomb repulsion.
$\longrightarrow$ We would generally expect that Z would be somewhat less than $A/2$ and N would be more than $A/2$.
The relative size of these two terms is not the same for all A. The asymmetry term falls of as $1/A$ while the Coulomb terms falls of only as $1/A^{1/3}$ $\Longrightarrow$ the latter becomes more important as A increases.
Conversely, for small A, particularly as $a_c \approx 0.7 MeV \ll a_a \approx 23.2 MeV$, the Coulomb term has little effect
$\Longrightarrow$ we expect $Z \approx A/2$ for small A but $Z < A/2$ for large A.

β- Stability curve



The actual stable and observed nuclei which form the beta-stability valley.
The line of the most stable nuclei is call the beta-stability curve.
Note, there can be several stable nuclei for a given A.
Nuclei lying on the beta-stability curve are often studied.

Magic numbers and Mirror nuclei

Magic numbers

Particular values of Z and N for which the nuclei had a higher binding energy than would be expected. These strongly bound states occur when Z or N have one of a set of so-called “magic numbers”:

$$2, 8, 20, 28, 50, 82, 126$$

Some nuclei have both Z and N at magic numbers, are called doubly-magic and are correspondingly even more strongly bound.

Mirror nuclei

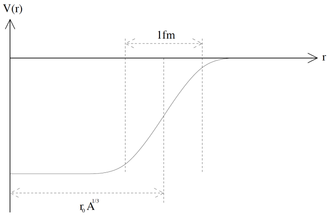
Mirror nuclei are pairs of isobars (same A), in which the proton number of one of the nuclides equals the neutron number of the other and vice versa.

Shell Model

The semi-empirical mass formula does a good job of describing trends but not the non-smooth behaviour of the binding energy. For this, we need to go to a very different model of the nucleus, which is based on quantum energy levels.
The existence of these magic numbers can be explained in terms of shell model.
The existence of these discrete energy levels for the nucleons in the nucleus is reminiscent of the atomic electron cloud. The electrons move in the atom in a central Coulombic potential emanating from the atomic nucleus. In the nucleon, on the other hand, the nucleons move inside a (mean field) potential produced by the other nucleons. In both cases discrete energy levels arise which are filled up according to the dictates of the Pauli principle.
For this we need first to introduce a suitable global nuclear potential.

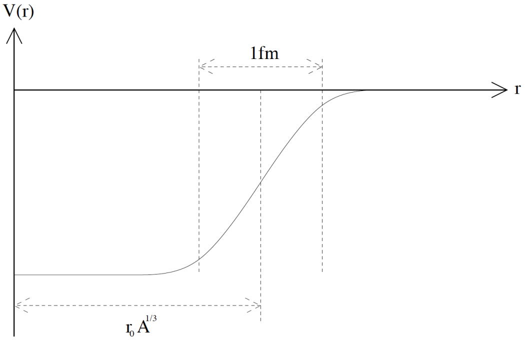
Nuclear Potential

The short range of nuclear force means a nucleon is bound to all its nearest neighbours by an equal contribution to the binding energy for each nucleon. Inside the nucleus, the number of nearest neighbours is equal in all directions so the net force on any nucleon is in fact zero. Thus the effective potential is constant within the nucleus and the constant value must be negative to keep the nucleon bound.



Outside the nucleus, more than a few fermis away, the short range nuclear force will have died off, so again there will be no force and hence a constant effective potential, which we can take as zero. The nucleons near the surface have only nearest neighbour forces into the nucleus as they are missing the nearest neighbours outside. Hence, they do have a net inwards force and so a rising potential as the radius increases. This change to the potential takes place over a distance of order the nuclear force, so around $1fm$

Saxon-Woods potential

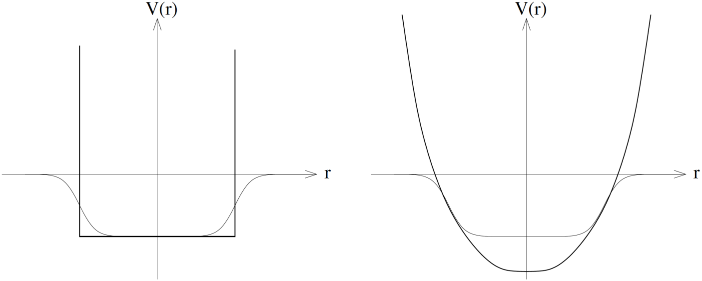


This is called the Saxon-Woods potential and is often mathematically expressed as

$$V(r) = -\frac{V_0}{1 + e^{(r-a)/d}}$$

where a is the radius of the nucleus, $= R_0 A^{1/3}$ and $d = 1 fm$

To get a feel for the results of the nuclear potential, Saxon-Woods potential we can look at some simpler cases, such as an infinite square well or a simple harmonic oscillator.



Woods-Saxon nuclear potential can also be written as:

$$V_{\mathrm{centre}}(r) = \frac{-V_0}{1 + \mathrm{e}^{(r-R)/a}}$$

Nuclear Energy Levels

Eigenstates of the nuclear potential. The wave function of the particles in the nuclear potential can divided into two parts: a radial one $R_{n\ell}(r)$, which only depends upon the radius, and a part $Y_\ell^m(\theta,\varphi)$ which only depends upon the orientation (this division is possible for all spherically symmetric potentials; e.g., atoms or quarkonium). The spectroscopic nomenclature for quarkonium is also employed for the quantum numbers here (see p. 174):

$$n\ell \quad \text{with} \begin{cases} n = 1,2,3,4,\cdots & \text{number of nodes} + 1 \\ \ell = \mathrm{s},\mathrm{p},\mathrm{d},\mathrm{f},\mathrm{g},\mathrm{h},\cdots & \text{orbital angular momentum}\,. \end{cases}$$

The energy is independent of the m quantum number, which can assume any integer value between $\pm\ell$. Since nucleons also have two possible spin directions, this means that the $n\ell$ levels are in fact $2\cdot(2\ell+1)$ times degenerate.

$$E_{\text{harm. osc.}} = (N+3/2)\cdot\hbar\omega = (N_x+N_y+N_z+3/2)\cdot\hbar\omega$$

where N is related to n and ℓ by

$$N = 2(n-1)+\ell\,.$$

Nuclear Energy Levels

The first three magic numbers (2, 8 and 20) can then be understood as nucleon numbers for full shells:

N	0	1	2	2	3	3	4	4	4	$\cdots$
$n\ell$	1s	1p	1d	2s	1f	2p	1g	2d	3s	$\cdots$
Degeneracy	2	6	10	2	14	6	18	10	2	$\cdots$
States with $E \leq E_{n\ell}$	2	8	18	20	34	40	58	68	70	$\cdots$

This simple model does not work for the higher magic numbers. For them it is necessary to include spin-orbit coupling effects which further split the *nl* shells.

Nuclear Energy Levels

These levels can be calculated more easily and look like the ones on next slide.

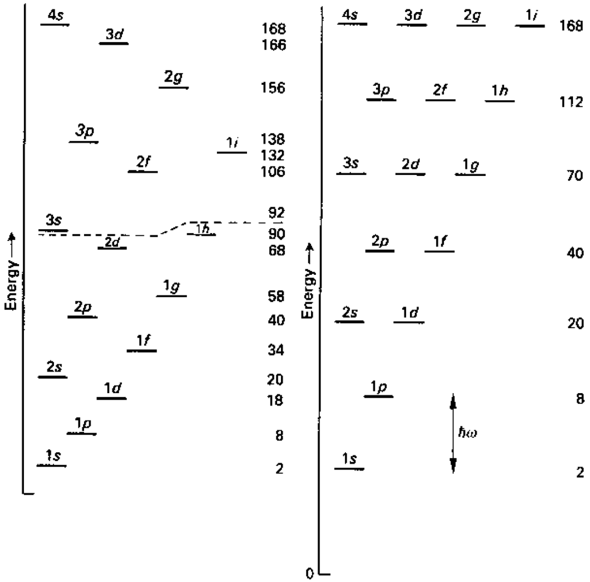
Each state has $2l + 1$ values of l_z and due to the nucleon spin, each can take two protons (and also two neutrons) in the two s_z states.

Hence, the number of protons (or neutrons) in an l state is 2(2l + 1) = 4l + 2

the first magic number of 2 corresponds to filling the first state in both cases. The next magic number is 8, which is the total number of nucleons which fills the first two states, again in either case.

The other numbers given by completing the levels are shown in the diagrams above. They both give 20 but then start to disagree with the measured values for the magic numbers. Hence, we can reproduce the first few but not the higher values.

Nuclear Energy Levels



Spin-Orbit Coupling

$$V(r) = V_{\text{centr}}(r) + V_{\ell s}(r)\,\frac{\langle \boldsymbol{\ell s} \rangle}{\hbar^2}\,. \tag{17.25}$$

The combination of the orbital angular momentum $\boldsymbol{\ell}$ and the nucleon spin $\boldsymbol{s}$ leads to a total angular momenta $j\hbar = \ell\hbar \pm \hbar/2$ and hence to the expectation values

$$\frac{\langle \boldsymbol{\ell s} \rangle}{\hbar^2} = \frac{j(j+1) - \ell(\ell+1) - s(s+1)}{2} = \begin{cases} \ell/2 & \text{for } j = \ell + 1/2 \\ -(\ell+1)/2 & \text{for } j = \ell - 1/2\,. \end{cases} \tag{17.26}$$

This leads to an energy splitting $\Delta E_{\ell s}$ which linearly increases with the angular momentum as

$$\Delta E_{\ell s} = \frac{2\ell + 1}{2} \cdot \langle V_{\ell s}(r) \rangle\,. \tag{17.27}$$

It is found experimentally that $V_{\ell s}$ is negative, which means that the $j = \ell + 1/2$ is always below the $j = \ell - 1/2$ level, in contrast to the atomic case, where the opposite occurs.

Usually the total angular momentum quantum number $j = \ell \pm 1/2$ of the nucleon is denoted by an extra index. So, for example, the 1f state is split into a $1\mathrm{f}_{7/2}$ and a $1\mathrm{f}_{5/2}$ state. The $n\ell_j$ level is $(2j+1)$ times degenerate.

Spin-Orbit Coupling: Expectation values

$$\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$$

so squaring gives

$$j^2 = l^2 + s^2 + 2\boldsymbol{l}.\boldsymbol{s}$$

Rearranging, then

$$\boldsymbol{l}.\boldsymbol{s} = \frac{1}{2}\left[j^2 - l^2 - s^2\right]$$

In terms of eigenvalues, this is

$$\langle \boldsymbol{l}.\boldsymbol{s} \rangle = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - s(s+1)\right]$$

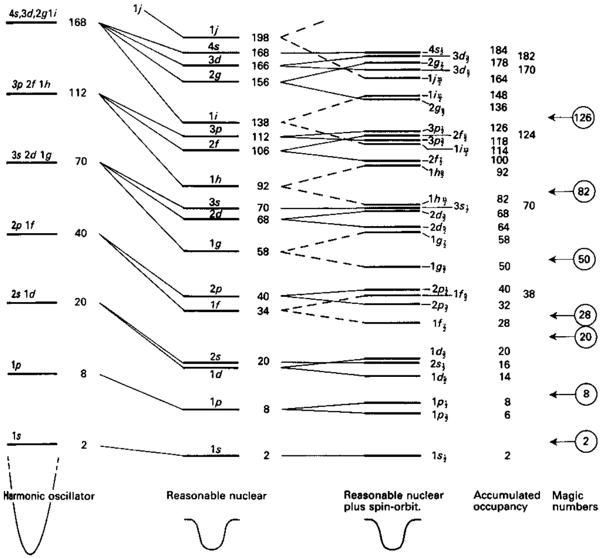
showing that this term does indeed depend on the value of j. Since $j = l \pm 1/2$, then for $l + 1/2$, this gives

$$\langle \boldsymbol{l}.\boldsymbol{s} \rangle = \frac{\hbar^2}{2}\left[(l+1/2)(l+3/2) - l(l+1) - s(s+1)\right] = \frac{\hbar^2}{2}\left[l^2 + 2l + 3/4 - l^2 - l - 3/4\right] = \frac{\hbar^2}{2} l$$

while for $l - 1/2$, it gives

$$\langle \boldsymbol{l}.\boldsymbol{s} \rangle = \frac{\hbar^2}{2}\left[(l-1/2)(l+1/2) - l(l+1) - s(s+1)\right] = \frac{\hbar^2}{2}\left[l^2 - 1/4 - l^2 - l - 3/4\right] = -\frac{\hbar^2}{2}(l+1)$$

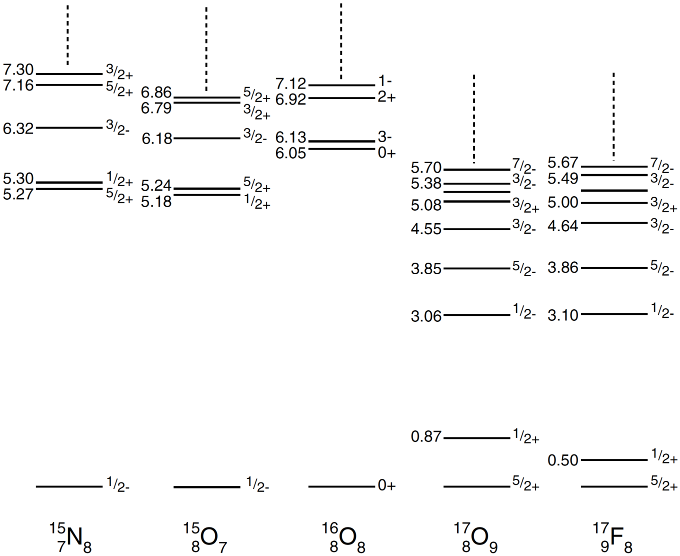
The effect of applying this splitting to the Saxon-Woods potential is shown below:



Mirror Nuclei

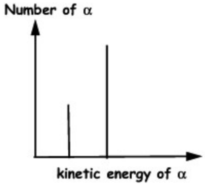
- ^{15}N and ^{15}O nuclei are so-called **mirror nuclei**, i.e., the neutron number of the one is equal to the proton number of the other and vice versa. Their spectra are exceedingly similar in terms of where the levels are.
- The small differences in the spectra can be understood as electromagnetic effects. While the energy levels of ^{16}O do not resemble those of its neighbours, the ^{17}O and ^{17}F nuclei are, once again, mirror nuclei and have very similar excitation spectra. It is striking that the nuclei with mass numbers 15 and 16 require much more energy to reach their first excited states than do those with mass number 17.
- These spectra can be understood inside the shell model. The ^{16}O nucleus possesses 8 protons and 8 neutrons. In the ground state the $1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$ proton and neutron shells are fully occupied and the next highest shells, $1d_{5/2}$, are empty.

Mirror Nuclei



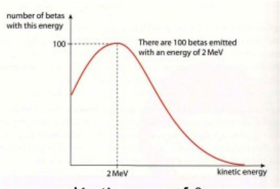
Natural radioactive decays is when a nucleus lose energy by emitting radiation. This happens when the nucleus can change to a more stable nucleus (higher binding energy per nucleon) by sending out the radiation. So a nucleus will decay if there is a set of particles with lower total mass that can be reached by any radioactive decay process.

α – radiation



This radiation has constant energy values.

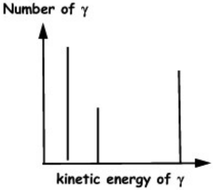
β – radiation



kinetic energy of β

This radiation has not constant energy values because the kinetic energy is shared between the β and the ν.

γ – radiation



This radiation has constant energy values.

Decay constants. The probability per unit time for a radioactive nucleus to decay is known as the *decay constant* λ. It is related to the *lifetime* τ and the *half life* $t_{1/2}$ by:

$$\tau = \frac{1}{\lambda} \qquad \text{and} \qquad t_{1/2} = \frac{\ln 2}{\lambda}\ . \tag{3.2}$$

The measurement of the decay constants of radioactive nuclei is based upon finding the *activity* (the number of decays per unit time):

$$A = -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N \tag{3.3}$$

where N is the number of radioactive nuclei in the sample. The unit of activity is defined to be

$$1 \text{ Bq [Becquerel]} = 1 \text{ decay /s.} \tag{3.4}$$

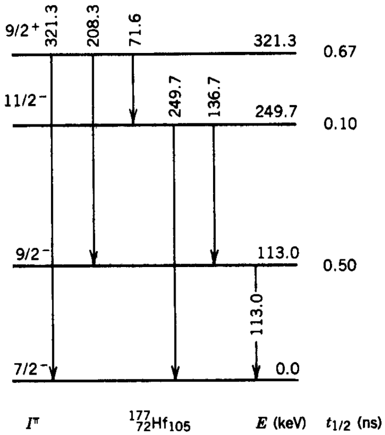
For short-lived nuclides, the fall-off over time of the activity:

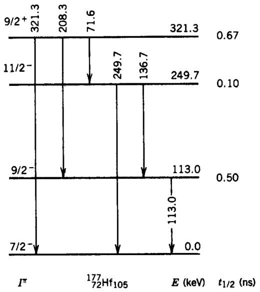
$$A(t) = \lambda N(t) = \lambda N_0\, \mathrm{e}^{-\lambda t} \qquad \text{where} \quad N_0 = N(t=0) \tag{3.5}$$

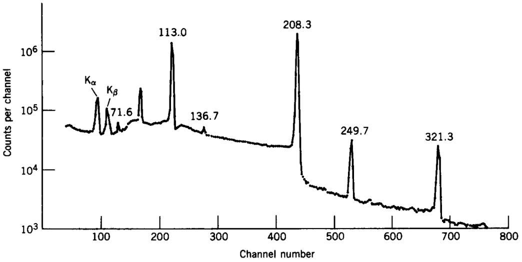
may be measured using fast electronic counters. This method of measuring is not suitable for lifetimes larger than about a year. For longer-lived nuclei both the number of nuclei in the sample and the activity must be measured in order to obtain the decay constant from (3.3).

γ decay

We now know that the gamma particle observed in radiation is simply a high energy photon, so γ decays are EM decays.
γ decay cannot change the nucleus values of Z or N through these decays. Hence, there is a very limited type of decay possible, specifically from an excited state to a ground state of the same nucleus. This is therefore the nuclear equivalent of atomic emission of light, but at much higher energies. In the same way, the energy differences between levels can be deduced from the energy spectrum of the photons seen.





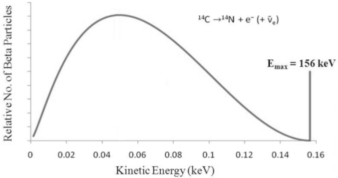


Being an electromagnetic decay, the de-excitations tend to happen reasonably fast, as long as there is not too big a change of angular momentum between the initial and final state. For photons of order 1 MeV between similar states, lifetimes tend to be around $10^{-16}s$, while for large changes in angular momentum $\Delta J = 4 or 5$, this can increase to $10^3 s$.

β decay: History

An outstanding puzzle was related to the β-decay process. The continuous energy distribution of beta decay electrons was a confusing experimental result in the 1920s. The energy distribution extends from zero to an upper limit (the endpoint energy) which is equal to the energy difference between the quantized initial and final nuclear states.

$$\beta \text{ decay} : n \rightarrow p + e^- + \bar{\nu_e}$$



This puzzle was solved by Pauli who suggested the existence of a new, very light uncharged and penetrating particle, the neutrino. In 1930 this was a revolutionary step. The neutrino carries the missing energy, Conservation of electric charge requires the neutrino to be electrically neutral and angular momentum conservation and spin statistics considerations in the decay process require the neutrino to behave like a fermion of spin 1/2.

β decay: Odd A

β-decay in odd mass nuclei. In what follows we wish to discuss the different kinds of β-decay, using the example of the $A = 101$ isobars. For this mass number, the parabola minimum is at the isobar $^{101}\mathrm{Ru}$ which has $Z = 44$. Isobars with more neutrons, such as $^{101}_{42}\mathrm{Mo}$ and $^{101}_{43}\mathrm{Tc}$, decay through the conversion:

$$\mathrm{n} \;\rightarrow\; \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}\,. \tag{3.7}$$

The charge number of the daughter nucleus is one unit larger than that of the the parent nucleus (Fig. 3.2). An electron and an e-antineutrino are also produced:

$$\begin{aligned} ^{101}_{42}\mathrm{Mo} &\rightarrow {}^{101}_{43}\mathrm{Tc} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}\,, \\ ^{101}_{43}\mathrm{Tc} &\rightarrow {}^{101}_{44}\mathrm{Ru} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}\,. \end{aligned}$$

β decay

Historically such decays where a negative electron is emitted are called β^--decays. Energetically, β^--decay is possible whenever the mass of the daughter atom $M(A, Z+1)$ is smaller than the mass of its isobaric neighbour:

$$M(A, Z) > M(A, Z+1)\,. \tag{3.8}$$

We consider here the mass of the whole atom and not just that of the nucleus alone and so the rest mass of the electron created in the decay is automatically taken into account. The tiny mass of the (anti-)neutrino ($<15\,\mathrm{eV}/c^2$) [PD98] is negligible in the mass balance.

β decay

Isobars with a proton excess, compared to $^{101}_{44}\mathrm{Ru}$, decay through proton conversion:

$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}\,. \tag{3.9}$$

The stable isobar $^{101}_{44}\mathrm{Ru}$ is eventually produced via

$$\begin{aligned} ^{101}_{46}\mathrm{Pd} &\rightarrow {}^{101}_{45}\mathrm{Rh} + \mathrm{e}^+ + \nu_\mathrm{e}\,, \quad \text{and} \\ ^{101}_{45}\mathrm{Rh} &\rightarrow {}^{101}_{44}\mathrm{Ru} + \mathrm{e}^+ + \nu_\mathrm{e}\,. \end{aligned}$$

Such decays are called β^+-decays. Since the mass of a free neutron is larger than the proton mass, the process (3.9) is only possible inside a nucleus. By contrast, neutrons outside nuclei can and do decay (3.7). Energetically, β^+-decay is possible whenever the following relationship between the masses $M(A, Z)$ and $M(A, Z-1)$ (of the parent and daughter atoms respectively) is satisfied:

$$M(A, Z) > M(A, Z-1) + 2m_\mathrm{e}\,. \tag{3.10}$$

This relationship takes into account the creation of a positron and the existence of an excess electron in the parent atom.

β decay: Odd A = 101 Isobars

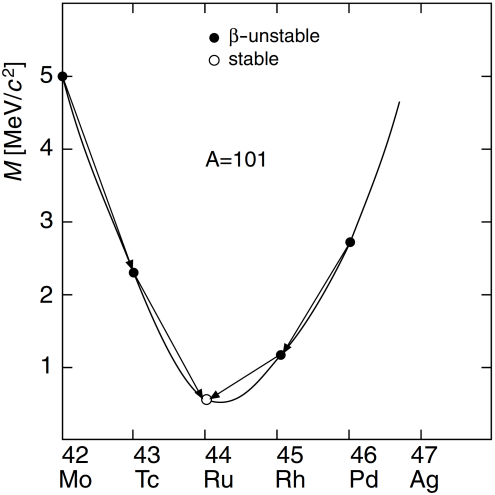
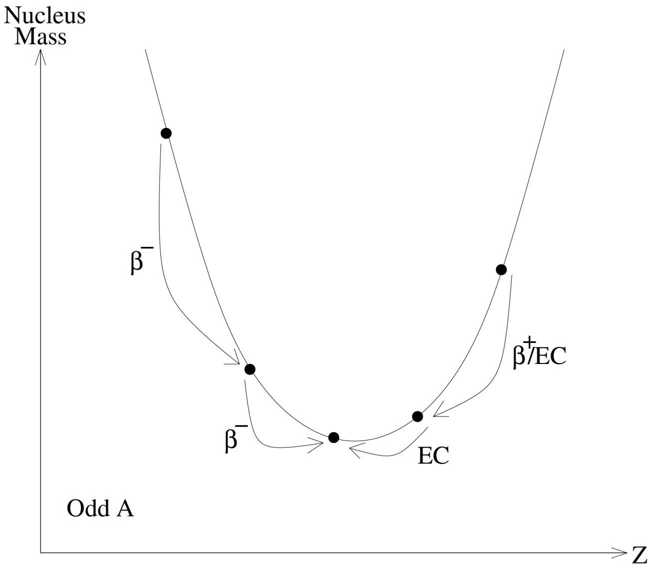


Fig. 3.2. Mass parabola of the $A = 101$ isobars (from [Se77]). Possible β-decays are shown by arrows. The abscissa co-ordinate is the atomic number, Z. The zero point of the mass scale was chosen arbitrarily.

β decay: Generic Odd A



β decay: Even A

$\boldsymbol{\beta}$-decay in even nuclei. Even mass number isobars form, as we described above, two separate (one for even-even and one for odd-odd nuclei) parabolas which are split by an amount equal to twice the pairing energy.

Often there is more than one β-stable isobar, especially in the range $A > 70$. Let us consider the example of the nuclides with $A = 106$ (Fig. 3.3). The even-even $^{106}_{46}$Pd and $^{106}_{48}$Cd isobars are on the lower parabola, and $^{106}_{46}$Pd is the stablest. $^{106}_{48}$Cd is β-stable, since its two odd-odd neighbours both lie above it. The conversion of $^{106}_{48}$Cd is thus only possible through a double β-decay into $^{106}_{46}$Pd:

$$^{106}_{48}\mathrm{Cd} \;\rightarrow\; ^{106}_{46}\mathrm{Pd} + 2\mathrm{e}^{+} + 2\nu_{\mathrm{e}}\,.$$

The probability for such a process is so small that $^{106}_{48}$Cd may be considered to be a stable nuclide.

Odd-odd nuclei always have at least one more strongly bound, even-even neighbour nucleus in the isobaric spectrum. They are therefore unstable. The only exceptions to this rule are the very light nuclei ${}^{2}_{1}$H, ${}^{6}_{3}$Li, ${}^{10}_{5}$B and ${}^{14}_{7}$N, which are stable to β-decay, since the increase in the asymmetry energy would exceed the decrease in pairing energy. Some odd-odd nuclei can undergo both β^{-}-decay and β^{+}-decay. Well-known examples of this are ${}^{40}_{19}$K (Fig. 3.4) and ${}^{64}_{29}$Cu.

β decay: Even A = 106 Isobars

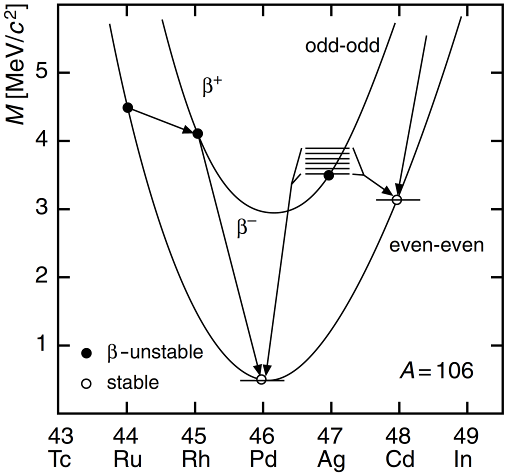
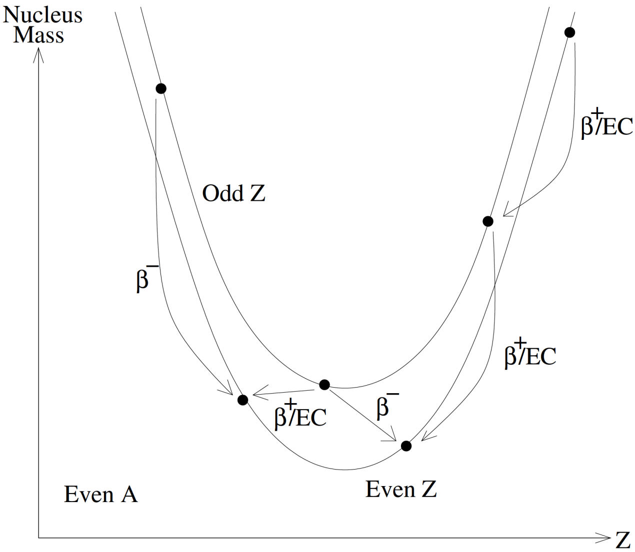


Fig. 3.3. Mass parabolas of the $A = 106$-isobars (from [Se77]). Possible β-decays are indicated by arrows. The abscissa coordinate is the charge number Z. The zero point of the mass scale was chosen arbitrarily.

β decay: Generic Even A



Electron capture

Electron capture. Another possible decay process is the capture of an electron from the cloud surrounding the atom. There is a finite probability of finding such an electron inside the nucleus. In such circumstances it can combine with a proton to form a neutron and a neutrino in the following way:

$$\mathrm{p} + \mathrm{e}^- \to \mathrm{n} + \nu_\mathrm{e}\,. \tag{3.11}$$

This reaction occurs mainly in heavy nuclei where the nuclear radii are larger and the electronic orbits are more compact. Usually the electrons that are captured are from the innermost (the "K") shell since such K-electrons are closest to the nucleus and their radial wave function has a maximum at the centre of the nucleus. Since an electron is missing from the K-shell after such a *K-capture*, electrons from higher energy levels will successively cascade downwards and in so doing they emit characteristic X-rays.

Electron capture

Electron capture reactions compete with β^+-decay. The following condition is a consequence of energy conservation

$$M(A,Z) > M(A,Z-1) + \varepsilon \; , \tag{3.12}$$

where ε is the excitation energy of the atomic shell of the daughter nucleus (electron capture always leads to a hole in the electron shell). This process has, compared to β^+-decay, more kinetic energy ($2m_\mathrm{e}c^2 - \varepsilon$ more) available to it and so there are some cases where the mass difference between the initial and final atoms is too small for conversion to proceed via β^+-decay and yet K-capture can take place.

Lifetimes. The lifetimes τ of β-unstable nuclei vary between a few ms and 10^{16} years. They strongly depend upon both the energy E which is released ($1/\tau \propto E^5$) and upon the nuclear properties of the mother and daughter nuclei. The decay of a free neutron into a proton, an electron and an antineutrino releases 0.78 MeV and this particle has a lifetime of $\tau = 886.7 \pm 1.9\,\mathrm{s}$ [PD98]. No two neighbouring isobars are known to be β-stable.[1]

A well-known example of a long-lived β-emitter is the nuclide ^{40}K. It transforms into other isobars by both β^-- and β^+-decay. Electron capture in ^{40}K also competes here with β^+-decay. The stable daughter nuclei are ^{40}Ar and ^{40}Ca respectively, which is a case of two stable nuclei having the same mass number A (Fig. 3.4).

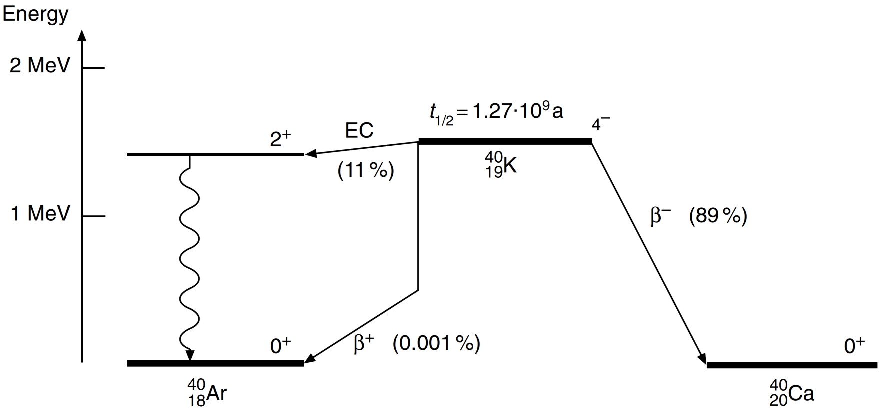
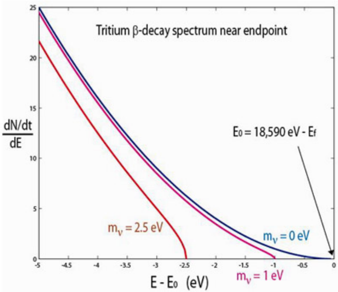


Fig. 3.4. The β-decay of ^{40}K. In this nuclear conversion, β^-- and β^+-decay as well as electron capture (EC) compete with each other. The relative frequency of these decays is given in parentheses. The bent arrow in β^+-decay indicates that the production of an e^+ and the presence of the surplus electron in the ^{40}Ar atom requires 1.022 MeV, and the remainder is carried off as kinetic energy by the positron and the neutrino. The excited state of ^{40}Ar produced in the electron capture reaction decays by photon emission into its ground state.

Neutrino mass

Note, electron capture is a two-body final state so the neutrino (and nucleus) have a single fixed energy of emission. In the other decays, the electron and positron energies are not unique. The spectrum goes from zero right up to the difference in mass of the initial and final nucleus, ignoring the neutrino mass. If, however, the neutrino has a non-zero mass, then the maximum energy of the electron or positron is more limited.



This has been used to try to measure the neutrino mass; the upper portion of the electron spectrum should be truncated by an amount which depends on the neutrino mass. The most studied beta decay for this purpose is tritium, which decays to helium-3, ${}^3_1H \rightarrow {}^3_2 He + e^- + \bar{\nu_e}$ with a very small energy release of only $Q = 18.6 keV$. The small energy release makes the observation of the effects of a non-zero neutrino mass easier to see. However, it is still a very hard experiment and while neutrino masses of more than $\approx 15\ eV/c^2$ have been excluded it is very hard to rule out neutrino masses at the level implied by oscillations.

Q-value : Definition

$$ {}^{A}_{Z}\mathrm{X}_{N} + {}^{A'}_{Z'}\mathrm{a}_{N'} \rightarrow {}^{A''}_{Z''}\mathrm{Y}_{N''} + {}^{A'''}_{Z'''}\mathrm{b}_{N'''}. $$

Conservation of total energy, which becomes

$$ T_{\mathrm{X}} + M'_{\mathrm{X}}.c^2 + T_{\mathrm{a}} + M'_{\mathrm{a}}.c^2 = T_{\mathrm{Y}} + M'_{\mathrm{Y}}.c^2 + T_{\mathrm{b}} + M'_{\mathrm{b}}.c^2. \tag{1.70} $$

with T_{i} the kinetic energy and $M'_{\mathrm{i}}.c^2$ the mass–energy. In the non-relativistic situation, the kinetic energy $T_{\mathrm{i}} = \frac{1}{2}M'_{\mathrm{i}}v_{\mathrm{i}}^2$. One defines the Q-value of a given reaction as

$$ \begin{aligned} Q &= \left(\sum M'_{\mathrm{i}}c^2 - \sum M'_{\mathrm{f}}c^2\right) \\ &= \left(M'_{\mathrm{X}} + M'_{\mathrm{a}} - M'_{\mathrm{Y}} - M'_{\mathrm{b}}\right)c^2. \end{aligned} \tag{1.71} $$

which can be rewritten using the kinetic energies as

$$ \begin{aligned} Q &= \sum T_{\mathrm{f}} - \sum T_{\mathrm{i}} \\ &= T_{\mathrm{Y}} + T_{\mathrm{b}} - T_{\mathrm{X}} - T_{\mathrm{a}}. \end{aligned} \tag{1.72} $$

The Q_{β^-} value for the transmutation is

$$^{A}_{Z}\mathrm{X}_{N} \rightarrow ^{A}_{Z+1}\mathrm{Y}_{N-1} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}, \tag{5.6}$$

$$\begin{aligned} Q_{\beta^-} &= T_{\mathrm{e}^-} + T_{\bar{\nu}_{\mathrm{e}}} \\ &= M'_{\mathrm{P}}c^2 - M'_{\mathrm{D}}c^2 - m_0 c^2. \end{aligned} \tag{5.7}$$

We can convert the nuclear masses into atomic masses, using the expression

$$M_{\mathrm{P}}c^2 = M'_{\mathrm{P}}c^2 + Zm_0c^2 - \sum_{i=1}^{Z} B_i, \tag{5.8}$$

where the B_i represents the binding energy of the ith electron. So, one obtains for the Q_{β^-} value the result (neglecting the very small *differences* in electron binding energy between the daughter and parent atoms)

$$Q_{\beta^-} = M_{\mathrm{P}}c^2 - M_{\mathrm{D}}c^2. \tag{5.9}$$

Thus, the β^- process is possible (exothermic with $Q_{\beta^-} > 0$) with the emission of an electron and an antineutrino with positive kinetic energy whenever $M_{\mathrm{P}} > M_{\mathrm{D}}$.

Q-value β^+

The Q_{β^+} value for the transmutation

$$^{A}_{Z}\mathrm{X}_N \rightarrow ^{A}_{Z-1}\mathrm{Y}_{N+1} + \mathrm{e}^+ + \nu_\mathrm{e}, \tag{5.10}$$

is

$$\begin{aligned} Q_{\beta^+} &= T_{\mathrm{e}^+} + T_{\nu_\mathrm{e}} \\ &= M'_\mathrm{P}c^2 - M'_\mathrm{D}c^2 - m_0c^2. \end{aligned} \tag{5.11}$$

Using the same method as in equation 5.8 to transform nuclear masses in atomic masses, one obtains the result

$$Q_{\beta^+} = M_\mathrm{P}c^2 - \left(M_\mathrm{D}c^2 + 2m_0c^2\right). \tag{5.12}$$

So, the β^+ process has a threshold of $2m_0c^2$ in order for the process to proceed spontaneously ($Q_{\beta^+} > 0$).

Q-value Electron Capture

A process, rather similar to β^+-decay by which the nuclear charge also decreases by one unit ($Z \to Z - 1$), is called electron capture, can also occur. An atomic electron is 'captured' by a proton, thereby transforming into a bound neutron and emitting a neutrino. This process leaves the final atom in an excited state, since a vacancy has been created in one of the inner electron shells. It is denoted by

$$ {}^{A}_{Z}\mathrm{X}_N + \mathrm{e}^- \to {}^{A}_{Z-1}\,\mathrm{Y}_{N+1} + \nu_{\mathrm{e}}, \tag{5.13}$$

and has a Q_{EC} value

$$Q_{\mathrm{EC}} = M_{\mathrm{P}}c^2 - \left(M_{\mathrm{D}}c^2 + B_n\right), \tag{5.14}$$

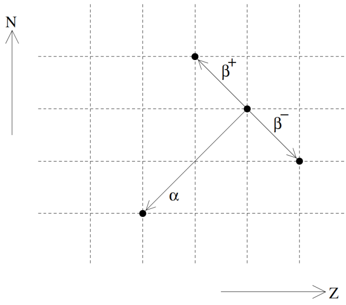
with B_n, the electron binding energy of the nth electron ($n =$K, $\mathrm{L_I}$, $\mathrm{L_{II}}$, $\mathrm{L_{III}}$, $\mathrm{M_I}$, ...) in the final atom. Here too, a constraint, albeit not very stringent, is present on the mass difference $M_{\mathrm{P}}c^2 - M_{\mathrm{D}}c^2$. Following the electron capture process, the vacancy created is very quickly filled as electrons from less bound orbitals make downward transitions thereby emitting characteristic X-rays. These processes will be discussed

α decay: Introduction

We have looked at gamma decays (due to the EM force) and beta decays (due to the weak force) and now will look at alpha decays, which are due to the strong/nuclear force. In contrast to the previous decays which do not change A, alpha decays happen by emission of some of the nucleons from the nucleus. Specifically, for alpha decay, an alpha particle, ${}^4_2\mathrm{He}$, is ejected so generically

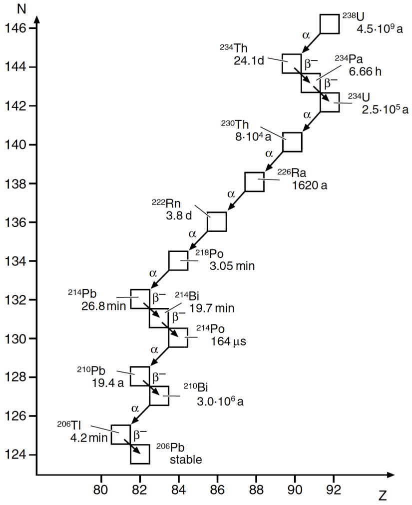
$$ {}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2\mathrm{He} $$

for some X and Y. Y clearly has a different number of nucleons to X. Compare how alpha and beta decays move the nuclei around in Z,N plane



Note that gamma decays cannot change Z, N or A.

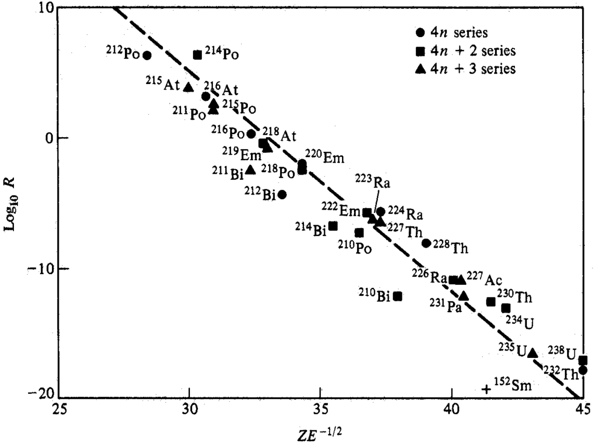
^{238}U decay chain in the N – Z plane



Alpha decays reduce Z and N equally, specifically reducing them each by two per decay. However, the heavy, large A starting nucleus will have $N > Z$ as that is what is needed to be near the beta-stability curve. Hence, a pure alpha decay sequence would leave a lower A nucleus with a higher and higher fraction of neutrons, whereas the beta-stability curve requires the fraction of neutrons to become lower as A is reduced. Hence, the beta decays bring the intermediate nuclei back closer to the beta- stability curve.

α decay rates

One obvious question is why do we see any of these sequences at all? This is a strong force decay and they have had around 5 billion years to decay.
The answer is that some of the lifetimes for these decays are billions of years, despite being due to the strong force.
The range of lifetimes for alpha decay is found to vary by over 25 orders of magnitude and this is effectively totally determined by the Z value and the amount of energy release, Q.



This extremely strong exponential dependence on $Z/\sqrt{Q}$ can be understood by considering the alpha decay process in more detail. To be emitted, the alpha particle has to form outside the nucleus

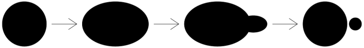


Figure 3.5 shows the potential energy of an α-particle as a function of its separation from the centre of the nucleus. Beyond the nuclear force range, the α-particle feels only the Coulomb potential $V_C(r) = 2(Z-2)\alpha\hbar c/r$, which increases closer to the nucleus. Within the nuclear force range a strongly attractive nuclear potential prevails. Its strength is characterised by the depth of the potential well. Since we are considering α-particles which are energetically allowed to escape from the nuclear potential, the total energy of this α-particle is positive. This energy is released in the decay.

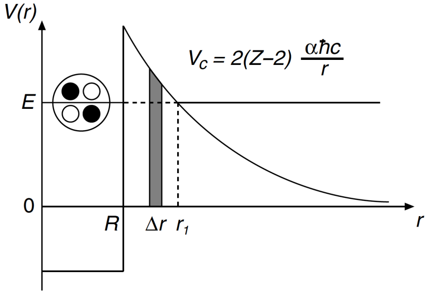


Fig. 3.5. Potential energy of an α-particle as a function of its separation from the centre of the nucleus. The probability that it tunnels through the Coulomb barrier can be calculated as the superposition of tunnelling processes through thin potential walls of thickness Δr (cf. Fig. 3.6).

If the energy release Q is large, then its energy will be above the maximum of the potential and it can decay very fast. However, the maximum of the potential will be:

$$V_{\max} \sim \frac{2(Z-2)e^2}{4\pi\epsilon_0 r_n} \sim \frac{2Ze^2}{4\pi\epsilon_0 r_0 A^{1/3}} = 2.4\frac{Z}{A^{1/3}}\ \mathrm{MeV}$$

For the nuclei which alpha decay, $Z = A^{1/3} \approx 15$ and so this barrier is several 10's of MeV, much larger than most observed alpha decay Q values.

The rate per nucleus R (equal to the inverse of the lifetime) is then expected to be

$$R = \frac{1}{\tau} = ae^{-2G}$$

so

$$\log R = \log a - (2\log e)G = \log a - 1.7\frac{Z}{\sqrt{Q}}$$

The plot shows a slope of -1.7 which agrees well with most of the measured values. Hence, tunneling is what causes the huge variation in lifetimes; for Q values which only differ by 1 MeV, a difference of 10^5 is seen.

The range of lifetimes for the α-decay of heavy nuclei is extremely large. Experimentally, lifetimes have been measured between 10 ns and 10^{17} years. These lifetimes can be calculated in quantum mechanics by treating the α-particle as a wave packet. The probability for the α-particle to escape from the nucleus is given by the probability for its penetrating the *Coulomb barrier* (the tunnel effect). If we divide the Coulomb barrier into thin potential walls and look at the probability of the α-particle tunnelling through one of these (Fig. 3.6), then the transmission T is given by:

$$T \approx \mathrm{e}^{-2\kappa\Delta r} \qquad \text{where} \quad \kappa = \sqrt{2m|E-V|}/\hbar\,, \tag{3.13}$$

and Δr is the thickness of the barrier and V is its height. E is the energy of the α-particle. A Coulomb barrier can be thought of as a barrier composed of

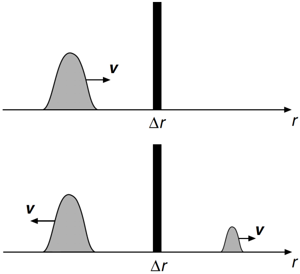


Fig. 3.6. Illustration of the tunnelling probability of a wave packet with energy E and velocity v faced with a potential barrier of height V and thickness Δr.

a large number of thin potential walls of different heights. The transmission can be described accordingly by:

$$T = \mathrm{e}^{-2G} \,. \tag{3.14}$$

The *Gamow factor* G can be approximated by the integral [Se77]:

$$G = \frac{1}{\hbar} \int_{R}^{r_1} \sqrt{2m|E-V|}\,\mathrm{d}r \approx \frac{\pi \cdot 2 \cdot (Z-2) \cdot \alpha}{\beta} \,, \tag{3.15}$$

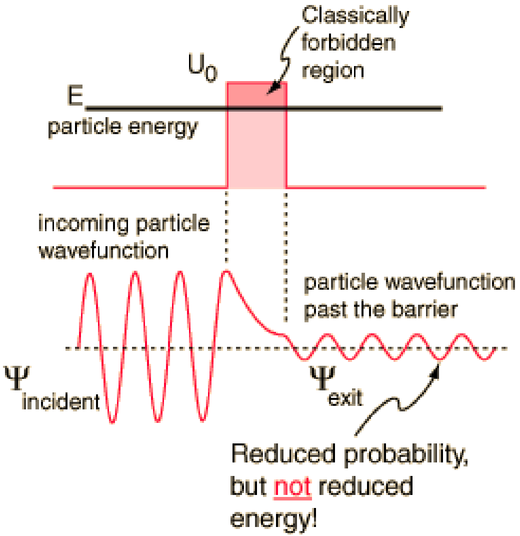
where $\beta = v/c$ is the velocity of the outgoing α-particle and R is the nuclear radius.

The probability per unit time λ for an α-particle to escape from the nucleus is therefore proportional to: the probability $w(\alpha)$ of finding such an α-particle in the nucleus, the number of collisions ($\propto v_0/2R$) of the α-particle with the barrier and the transmission probability:

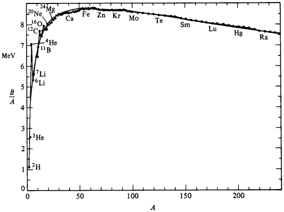
$$\lambda = w(\alpha)\,\frac{v_0}{2R}\,\mathrm{e}^{-2G} \,, \tag{3.16}$$

where v_0 is the velocity of the α-particle in the nucleus ($v_0 \approx 0.1\,c$). The large variation in the lifetimes is explained by the Gamow factor in the exponent: since $G \propto Z/\beta \propto Z/\sqrt{E}$, small differences in the energy of the α-particle have a strong effect on the lifetime.

α decay : QM



Nuclear Fusion & Fission



The binding energy per nucleon curve shows that the maximum occurs around $^{56}_{26}Fe$ and drops off to either side.

Hence, both small A and large A nuclei are less strongly bound per nucleon than medium A.

This means there will be some energy release if two small nuclei are combined into a larger one, a process called **nuclear fusion**.

Similarly, there will be an energy release if a large nucleus is split into two smaller parts, a process called **nuclear fission**.

Fission

We have seen alpha decay can be understood as **tunnelling through a Coulomb barrier** when the alpha has less energy than the barrier height.

Fission is the more general process of splitting a nucleus into two or more nuclear fragments. Although this clearly includes alpha decay, the term fission is more normally used to describe processes where the resulting nuclei are much **more even in size**.

Fission energetics

We have seen the maximum binding energy per nucleon is around $^{56}_{26}$Fe so we expect that fission is energetically possible for nuclei larger than around twice this size. It turns out that splitting a nucleus into two equal nuclei normally gives the largest energy release Q (as shown on the problem sheet). This is then

$$Q = m(Z,N) - 2m(Z/2,N/2) = 2B_E(Z/2,N/2) - B_E(Z,N)$$

This can be estimated from the semi-empirical mass formula. The volume terms clearly cancel, so ignoring the pairing term and approximating $Z(Z-1)$ to Z^2, then this is

$$Q = 2\left[-a_s\left(\frac{A}{2}\right)^{2/3} - a_c\frac{Z^2/4}{(A/2)^{1/3}} - a_a\frac{(N/2-Z/2)^2}{A/2}\right] + a_sA^{2/3} + a_c\frac{Z^2}{A^{1/3}} + a_a\frac{(N-Z)^2}{A}$$

This gives

$$\begin{aligned} Q &= a_sA^{2/3}\left[1-2\left(\frac{1}{2^{2/3}}\right)\right] + a_c\frac{Z^2}{A^{1/3}}\left[1-2\left(\frac{1}{2^{5/3}}\right)\right] + a_a\frac{(N-Z)^2}{A}\left[1-2\left(\frac{1}{2}\right)\right] \\ &= a_sA^{2/3}(1-2^{1/3}) + a_c\frac{Z^2}{A^{1/3}}(1-2^{-2/3}) \end{aligned}$$

Energy is released when $Q > 0$, meaning

$$a_c\frac{Z^2}{A^{1/3}}(1-2^{-2/3}) > a_sA^{2/3}(2^{1/3}-1)$$

or

$$\frac{Z^2}{A} > \frac{a_s(2^{1/3}-1)}{a_c(1-2^{-2/3})} = 0.702\frac{a_s}{a_c} \approx 18$$

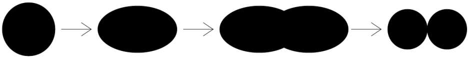
Fission energetics

For nuclei on the beta-stability curve this is actually satisfied for $A > 100$, which gives $Z > 42$. Hence, like alpha decay, fission is only energetically possible for heavy nuclei.

Note, fission involves a much bigger change to A than alpha decay and so gives a much bigger shift closer to $^{56}_{26}\mathrm{Fe}$ in the binding energy per nucleon plot. Hence, the energy releases in fission are significantly higher. We saw typical alpha decay energies are less than 10 MeV. Fission decays tend to release hundreds of MeV. This is one of the reasons why practical applications of nuclear energy use fission not alpha decay.

Spontaneous Fission

We can classically picture the process as the nucleus deforming and breaking up as follows:



One critical issue is whether it takes energy or not to deform the nucleus in this way. The two terms which influence this are again the surface term and the Coulomb term. We have seen that nucleons on the surface are less strongly bound, due to missing nearest neighbours. For a given volume, a sphere has the smallest surface area and hence the biggest binding energy. Hence, deforming from a sphere to an ellipsoid with the same volume increases the suface area and so requires energy. Explicitly, an ellipsoid can be defined in terms of a small deformation parameter δ, where the major axis is $r(1+\delta)$ and the two minor axes are $r/\sqrt{1+\delta}$. The surface area of the ellipsoid is then

$$4\pi r^2 \left(1 + \frac{2\delta^2}{5}\right)$$

and so the change to the nucleus energy is $a_s(A^{2/3})(2\delta^2/5)$. However, a deformed nucleus has the protons on average further away from each other and so the Coulomb energy is decreased. Hence, as a nucleus deforms, Coulomb energy is released. The approximate electrostatic energy of an ellipsoid for small deformations is

and so the change to the nucleus energy from this effect is $-a_c(Z^2/A^{1/3})(\delta^2/5)$. For the Coloumb term to be bigger, then

$$a_c \frac{Z^2}{A^{1/3}} \frac{\delta^2}{5} > a_s A^{2/3} \frac{2\delta^2}{5}$$

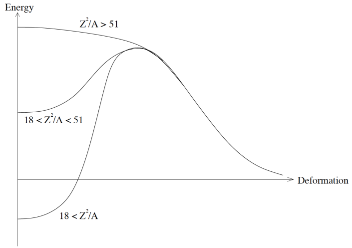
which means

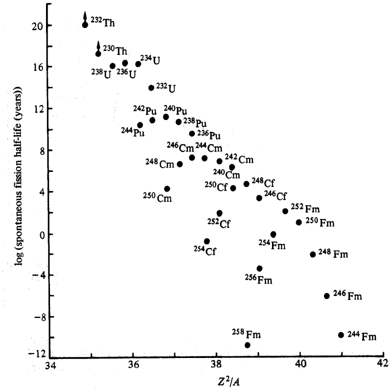
$$\frac{Z^2}{A} > \frac{2a_s}{a_c} \approx 51$$

For nuclei on the beta-stability curve, this corresponds to $A > 407$ which is $Z > 144$. For these very large nuclei, the deformation actually reduces the nucleus energy so there is no barrier to this happening. This means they can spontaneously decay through fission and will do so extremely fast, $\sim 10^{-20}$ s. This sets an absolute upper limit to the periodic table.

For nuclei intermediate between these two values, i.e. $18 < Z^2/A < 51$, then fission overall is energetically allowed but the deformation to start it requires energy and so, as for alpha decay, there is a potential barrier to tunnel through. In this case we cannot really consider one of the large daughter nuclei as forming and moving in a potential within the nucleus, so it is not so easy to draw such a potential well as for alpha decay. However a qualitative picture can be obtained by looking at how the energy needed to deform the nucleus depends on the deformation. We know that for nuclei in this range, it takes energy to deform the nucleus but when the daughter nuclei are separated enough that the nuclear force between them is small, then the Coulomb force dominates, as for alpha decay. Hence, a rough idea of the energy as it deforms is

Spontaneous Fission





The fact that nuclei in this intermediate range have to tunnel again gives a very strong lifetime range and exponential dependence on Z^2/A over 30 orders of magnitude.

Induced Fission

It is clear that fission for most of the heavy nuclei is difficult and hence slow. It can be speeded up enormously by exciting the nuclei. For practical uses of the fission energy, this is clearly essential. If enough energy can be added to a nucleus to raise its energy higher than the fission barrier, then it will fission within 10^{-20} s, the very fast rate associated with the strong force, as happens for the nuclei above the barrier, i.e. those with $Z^2/A > 51$.

There are several ways to excite nuclei to higher levels. The most obvious would be to bombard them with gamma radiation, as that is how excited nuclei can decay and indeed it is perfectly possible to excite them in this way. However, this is an EM reaction and so does not have as large a cross-section as a strong force reaction. In addition, there are no simple ways to make intense gamma sources in practise.

To have a high fission rate requires a strongly interacting particle, such as a proton, neutron or alpha. Of these, neutrons are special; they are neutral and so, unlike protons and alphas, are not repelled from the nucleus by their EM charge. To react via the nuclear force, protons and alphas would have to either have a high enough energy to overcome the Coulomb barrier or would have to tunnel through the barrier, drastically reducing the rate. Neutrons have no such problem and so can react at any energy effectively down to zero. Cross sections tend to be largest at low energies as, roughly speaking, there is more time to react. Neutron absorption dominates at such energies, leaving an excited nucleus. Clearly, any excited nucleus has some probability of gamma decaying to the ground state and this competes with the fission decay. The ratio of gamma to fission decays depends strongly on how far above the fission barrier the excited nucleus is.

Induced Fission

When a nucleus absorbs a neutron, the N and A values increase by one. Hence, if we have a nucleus ${}^{A}_{Z}X$ in which we want to induce fission, then we have to start with its isotope ${}^{A-1}_{Z}X$

$$ {}^{A-1}_{Z}X + n \rightarrow {}^{A}_{Z}X^{*} \rightarrow \text{fission} $$

Of course, the neutron has to have enough energy to excite the nucleus over the fission barrier. This depends on the relative binding energies of the two isotopes. The total input energy of the reaction is

$$ m(Z,N-1)c^2 + E_n = Zm_pc^2 + (N-1)m_nc^2 - B_{Ei} + m_nc^2 + T_n = Zm_pc^2 + Nm_nc^2 - B_{Ei} + T_n $$

where T_n is the neutron kinetic energy. The final nucleus has a energy in its ground state of

$$ m(Z,N)c^2 = Zm_pc^2 + Nm_nc^2 - B_{Ef} $$

Hence, neglecting the small nucleus recoil energy, the excited nucleus will be formed at an energy above the ground state given by the difference of these

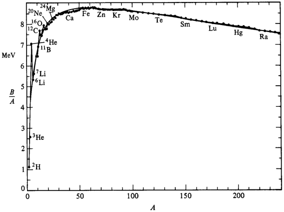
$$ \Delta E = B_{Ef} - B_{Ei} + T_n $$

Induced Fission

To fission, ΔE has to be greater than the fission barrier height. If the initial isotope is weakly bound and the final is strongly bound, so $B_{Ef} - B_{Ei}$ is large, then T_n can be small and even zero. Conversely, a small $B_{Ef} - B_{Ei}$ would need a large neutron energy with a subsequent reduction in cross section. The binding energy tends to be vary quite smoothly (except near magic numbers) in the semi-empirical mass formula with the exception of the pairing term. Clearly as A changes by one in neutron absorption, then one of the isotopes is even A and the other odd A. Hence, depending on whether the even A is even-even or odd-odd, the major part of the difference will

be given by $a_p/A^{1/2} \sim 2$ MeV. Examples of nuclei which can fission after absorbing even a zero energy neutron are the odd A nuclei $^{233}_{92}$U, $^{235}_{92}$U, $^{239}_{94}$Pu and $^{241}_{94}$Pu. These all have even Z and odd N; the extra neutron then makes these nuclei even-even and so have a large B_{Ef}. Even A nuclei which require a fast neutron include $^{232}_{90}$Th, $^{238}_{92}$U, $^{240}_{94}$Pu and $^{242}_{94}$Pu. In these cases, they are all even-even and so have a large B_{Ei} to start with.

Nuclear Fusion



The binding energy per nucleon curve shows that the maximum occurs around $^{56}_{26}Fe$ and drops off to either side.

It is the case that nuclei below the maximum can combine and release energy, a process called fusion.

Therefore, energy will be released through fusion. The generic fusion process is

$$^{A_X}_{Z_X}X + \, ^{A_Y}_{Z_Y}Y \to \, ^{A_X+A_Y}_{Z_X+Z_Y}F$$

To extract energy then the final nucleus has to have less total mass than the initial two nuclei. This means F must actually be in an excited state. The released energy will therefore come out when it decays. For example, it can gamma decay to release the extra energy

$$^{A_X}_{Z_X}X + \, ^{A_Y}_{Z_Y}Y \to \, ^{A_X+A_Y}_{Z_X+Z_Y}F^* \to \, ^{A_X+A_Y}_{Z_X+Z_Y}F + \gamma$$

We often don't need to consider F^* explicitly and indeed, often no coherent nucleus can be considered to be made.

The simplest case of such a reaction is the creation of a deuteron ($d = \, ^2_1\mathrm{H}$) from a proton and neutron

$$p + n \to d + \gamma$$

where, for $E_p \approx E_n \approx 0$, the emitted photon energy is $E_\gamma = Q = (m_p + m_n - m_d)c^2 = 2.2$ MeV. This is clearly an electromagnetic reaction and so in general does not have such a large cross section as a strong reaction.

Other possibilities for the energy to be released are to emit protons or neutrons in a strong interaction, e.g. An example of this is the so-called "D-T" reaction of a deuteron and tritium ($t = \, ^3_1\mathrm{H}$) to make helium and a neutron

$$d + t \to \, ^4_2\mathrm{He} + n$$

Helium

Let's look at the binding energy per nucleon plot again in more detail. A clear feature is the large value for $^4_2$He, particularly compared with the values just above it. $^4_2$He is doubly magic and stands out as being much more strongly bound, with a value of $B_E/A \approx 7.1$ MeV, than its neighbours. It is, in fact, more strongly bound than all the higher nuclei until $^{12}_6$C. Indeed there are no $A = 5$ or $A = 8$ stable nuclei because $^4_2$He is so stable. $^5_2$He and $^5_3$Li very rapidly decay by nucleon emission

$$^5_2\text{He} \rightarrow \ ^4_2\text{He} + n, \qquad ^5_3\text{Li} \rightarrow \ ^4_2\text{He} + p$$

with lifetimes of order 10^{-21} s, while $^8_4$Be spontaneously fissions to two $^4_2$He nuclei with a lifetime of order 10^{-16} s. If $^4_2$He had a lower binding energy, these reactions would not be energetically possible.

The maximum value of the binding energy per nucleon is around 8.7 MeV for nuclei close to iron. If we create $^4_2$He in fusion as a first step to building up to iron, we already get over 80% of the maximum possible energy release. Also, the next step would require combining $^4_2$He nuclei into the next nucleus with a higher binding energy, which is $^{12}_6$C. This requires three $^4_2$He nuclei and is very hard to do; reacting three nuclei at once is almost impossible and we have already seen the most obvious intermediate nucleus, $^8_4$Be, decays very rapidly. Even if these problems can be overcome, this reaction only yields another 0.6 MeV per nucleon.

Helium

Hence, all practical applications of fusion concentrate on combining hydrogen isotopes into ${}^4_2$He. This also has the advantages that the Coulomb barriers are smaller for these nuclei and that hydrogen and deuterium are readily available. Deuterium occurs naturally in 0.015% of water which is not a large fraction but water is clearly plentiful so the supply is enormous. Also, unlike uranium, deuterium is relatively easy to separate from hydrogen as the masses differ by a factor of two, not 1%. The most obvious reaction to make helium would be using two deuterons

$$d + d \to {}^4_2\mathrm{He} + \gamma, \qquad Q = 23.8 \text{ MeV}$$

but although the energy release is large, this is an EM reaction with a correspondingly low cross section. The more common reactions are

$$d + d \to {}^3_2\mathrm{He} + n, \qquad Q = 3.3 \text{ MeV}$$

and

$$d + d \to {}^3_1\mathrm{H} + p, \qquad Q = 4.0 \text{ MeV}$$

Helium

This final reaction produces tritium, which is the third isotope of hydrogen. Tritium beta decays to ${}^3_2\mathrm{He}$ with a lifetime of 17.7 years so none is found naturally and it has to be manufactured. This can be done from the above or by using lithium reactions

$$ {}^6_3\mathrm{Li} + n \to {}^4_2\mathrm{He} + t, \qquad {}^7_3\mathrm{Li} + n \to {}^4_2\mathrm{He} + t + n $$

As previously mentioned, tritium also reacts with a deuteron (the D-T reaction) to produce helium through a strong interaction

$$ d + t \to {}^4_2\mathrm{He} + n, \qquad Q = 17.6 \text{ MeV} $$

This releases a lot of energy as the helium is strongly bound and happens to have a large cross section, making it good for practical applications. The main disadvantage is that tritium is needed, which firstly must be manufactured and secondly must be replenished as it decays. Also, the neutron produced takes more than half the energy and extracting that for power uses is not straightforward.